A unified theory for optimal feedforward torque control of anisotropic synchronous machines

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Outline

1. MSE Group “Control of Renewable Energy Systems (CRES)”

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1. MSE Group “Control of Renewable Energy Systems (CRES)”
   - Team
   - Projects
   - Teaching
   - Research
Group “Control of renewable energy systems (CRES)”

Team (7 PhD candidates, 1 Post-Doc, 3 PhD collaborations)

H. Eldeeb, M.Sc. (CRES, 09/2015)
C. Dirscherl, M.Sc. (CRES, 01/2014)
C. Hackl, Dr.-Ing. (CRES, 01/2014)
J. Kullick, M.Sc. (CRES, 10/2015)
K. Schechner, M.Sc. (CRES, 01/2014)
M. Landerer, M.Sc. (CRES, 06/2016)

External PhD candidates

Z. Zhang, Dr.-Ing. (CRES/EAL, 07/2015)
A. Birda, M.Sc. (BMW, 04/2016)
F. Bauer, M.Sc. (EAL)
M. Abdelrahem, M.Sc. (EAL)
A. Ayad, M.Sc. (EAL)

... and collaborations with EAL (Prof. Kennel)
Group “Control of renewable energy systems (CRES)”

Research projects (all related to the electrical system)

- **Large-scale WTS**
  - Efficiency+Reliability

- **Small-scale WTS**
  - Reluctance SM

- **Model predictive control for RES**
  - Real-time applicability

- **Airborne Wind Energy**
  - Fault-tolerant control

- **Geothermal energy**
  - Fault-tolerant control

- **Wave energy (SinnPower)**
  - Efficiency+Reliability

- **Electric vehicles (BMW)**
  - Efficiency+Reliability

- **Power systems**
  - Four-wire system
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Teaching

- Lectures, tutorials, practical courses & student projects
  - “Regelung von regenerativen Energiesystemen” (WS, 3SWS with C. Dirscherl & K. Schechner)
  - “Non-identifier based adaptive control in mechatronics” (SS, 4SWS)
  - Supervised student projects: more than 50 (all CRES members, since 2014)

- Workshops
  - H2020 ITN Workshop “Electrical Drives and Predictive Control”: 15 (with M. Diehl)
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Research focus and publications: **Electrical** components of e.g. large-scale wind turbine systems

- **Generator current control**
  - parameter-free [14]
  - model-based [8, 15, 16, 23]
  - MTPA, MTPV [1, 2, 24–26]

- **Speed control**
  - encoderless [13, 17, 18]
  - parameter-free [19, 20]

- **Impacts of non-ideal torque control** [10, 21, 22]

- **Machine topologies**
  - PMSG [8, 10, 11]
  - DFIG [9, 12, 13]
  - RSM [14–16]

- **Dynamic power flow** [8, 9]

- **Modeling & control WTS (chapter)** [8]

- **Online parameter estimation DFIG** [12]

- **Converter topologies**
  - Two-level converters [8, 23, 27–30]
  - Three-level NPC converters [33–36]

- **Grid current control**
  - RL filter [8, 27, 28]
  - LCL filter [23, 31, 32]

- **Grid faults** [31, 32]

- **Grid synchronization**
  - PLL [8]
  - geberlos [33, 34]

- **Power control** [8, 33]

- **Converter topologies**
  - Two-level converters [8, 23, 27–30]
  - Three-level NPC converters [33–36]

- **Further contributions**
  - Dynamic friction modeling (chapter) [37]
  - Non-identifier based adaptive control in mechatronics (monograph) [38]
  - Airborne wind energy (chapters) [30, 39, 40]
A unified theory for optimal feedforward torque control of anisotropic synchronous machines [1–4]

- Motivation
- Problem statement and proposed solution
- Operation management and operation strategies
- Analytical solution
- Implementation
- Conclusion
Motivation

Synchronous machines: Widely-used, compact and efficient actuators

- traction drives (e.g. electric vehicles or trains)
- air conditioning/fans
- pumps
- conventional power plants (e.g. hydro power plants)
- renewable energy systems (e.g. direct-drive wind turbine systems)
Motivation

Examples of anisotropic synchronous machines [41] with "saliency ratio" $L_d^s/L_q^s \neq 1$

$$m_m(i_s^d, i_s^q) = \frac{3}{2} n_p \left[ \psi_{pm} i_s^q + (L_s^d - L_s^q) i_s^d i_s^q + L_m \left( (i_s^q)^2 - (i_s^d)^2 \right) \right]$$

- Magnetic torque
- Reluctance torque
- Cross-coupling torque
Motivation

Optimal feedforward torque control problem: **Isotropic machine**

\[
m_{m, \text{ref}} = m_m(i_s^d, i_s^q) = \frac{3}{2} n_p \psi_{pm} i_s^q \\
\Rightarrow i_s^q, \text{ref} = \frac{2 m_{m, \text{ref}}}{3 n_p \psi_{pm}}
\]
Motivation

Optimal feedforward torque control problem: **Anisotropic machine**

\[
m_{m,\text{ref}} = m_m(i_d, i_q) = \frac{3}{2} n_p \left[ \psi_{pm} i_q + (L_d^q - L_s^q) i_d i_q + L_m((i_s^q)^2 - (i_s^d)^2) \right]
\]
Motivation

Optimal feedforward torque control problem: Goal

- Numerical solutions and/or look-up tables (but: limited storage, accuracy, real-time applicability)
- Analytical solutions (some do exist but impose simplifying assumptions such as $R_s = 0$ and/or $L_m = 0$)
Problem statement and proposed solution
Considered machines: Anisotropic synchronous machines (steady-state model)

\[
\begin{align*}
\mathbf{u}_s^k (i_s^k, \omega_k) =: & \left( \begin{array}{c} u_s^d(i_s^k, \omega_k) \\ u_s^q(i_s^k, \omega_k) \end{array} \right) \\
\mathbf{u}_s(i_s^k) =: & \left( \begin{array}{c} i_s^d \\ i_s^q \end{array} \right) \\
\mathbf{J} =: & \left( \begin{array}{c} \psi_s^d(i_s^k) \\ \psi_s^q(i_s^k) \end{array} \right) \\
\mathbf{J}\psi_s^k(i_s^k) =: & \psi_s^k(i_s^k) \\
m_m(i_s^k) =: & \frac{3}{2} n_p \left( i_s^k \right)^\top \mathbf{J}\psi_s^k(i_s^k) \\
& = \frac{3}{2} n_p \left[ \psi_{pm}^k i_s^q + (L^d_s - L^q_s) i_s^d i_s^q + L_m \left( (i_s^q)^2 - (i_s^d)^2 \right) \right] \\
\end{align*}
\]

Assumption (Affine flux linkage (at least locally))

\[
\psi_s^k(i_s^k) = \left[ \begin{array}{c} L^d_s \\ L_m \\ L^q_s \end{array} \right] \left( \begin{array}{c} i_s^k \\ \psi_{pm} \end{array} \right) + \left( \begin{array}{c} \psi_{pm} \end{array} \right) \text{ with } L_s^k = (L_s^k)^\top > 0
\]
Problem statement and proposed solution

Feedforward torque control: Optimization problem(s) with multiple constraints

\[
\begin{align*}
\text{max} & - f(i_s^k) \quad \text{subject to} \\
& i_s^k \in \mathbb{S} \\
& \|i_s^k\|^2 \leq \hat{i}_{\max}^2, \quad \text{(current circular area)} \\
& \|u_s^k(i_s^k, \omega_k)\|^2 \leq \hat{u}_{\max}^2, \quad \text{(voltage elliptical area)} \\
& |m_m(i_s^k)| \leq |m_m, \text{ref}|, \quad \text{and} \\
& \text{sign}(m_m, \text{ref}) = \text{sign}(m_m(i_s^k)).
\end{align*}
\]

e.g. \(- f(i_s^k) = -\|i_s^k\|^2 \) (minimize copper losses) \(\implies i_s^{k,\text{ref}} \) at \(\star\)
Problem statement and proposed solution

Proposed solution: Implicit formulation (quadrics), optimization and intersection points

Step 1: Derivation of quadrics $Q_A(i_s^k) := (i_s^k)^T A i_s^k + 2a^T i_s^k + \alpha$:

- Current circular area: $(i_s^d)^2 + (i_s^q)^2 \leq \dot{i}_{\text{max}}^2 \iff \|i_s^k\|^2 = (i_s^k)^T I_2 i_s^k \leq \dot{i}_{\text{max}}^2$

  $$\implies \mathbb{I}(\dot{i}_{\text{max}}) := \{ i_s^k \in \mathbb{R}^2 \mid (i_s^k)^T I_2 i_s^k - \dot{i}_{\text{max}}^2 \leq 0 \}$$

- Voltage elliptical area: $(u_s^d)^2 + (u_s^q)^2 \leq \dot{u}_{\text{max}}^2 \iff \|u_s^k(i_s^k, \omega_k)\|^2 = \ldots \leq \dot{u}_{\text{max}}^2$

  $$\implies \mathbb{V}(\omega_k, \dot{u}_{\text{max}}) := \{ i_s^k \in \mathbb{R}^2 \mid (i_s^k)^T V(\omega_k) i_s^k + 2 v(\omega_k)^T i_s^k + v(\omega_k, \dot{u}_{\text{max}}) \leq 0 \}$$

- Reference torque hyperbola: $m_m(i_s^k) = \frac{3}{2} n_p (i_s^k)^T J \psi_s^k (i_s^k) \equiv m_m,_{\text{ref}} \iff \ldots$

  $$\implies \mathbb{T}(m_m,_{\text{ref}}) := \{ i_s^k \in \mathbb{R}^2 \mid (i_s^k)^T T i_s^k + 2 t^T i_s^k - m_m,_{\text{ref}} = 0 \}$$

...
Problem statement and proposed solution

Proposed solution: Implicit formulation (quadrics), optimization and intersection points (cont’d)

Step 2: Optimization problem with equality constraint

\[
\begin{align*}
\max_{\mathbf{i}_s^k} & \quad \left( (\mathbf{i}_s^k)^\top A \mathbf{i}_s^k + 2a^\top \mathbf{i}_s^k + \alpha \right) \quad \text{s.t.} \quad \left( (\mathbf{i}_s^k)^\top B \mathbf{i}_s^k + 2b^\top \mathbf{i}_s^k + \beta \right) = 0 \\
\quad =: Q_A(\mathbf{i}_s^k) \quad \quad & \quad =: Q_B(\mathbf{i}_s^k)
\end{align*}
\]

\[\implies \text{Hyperbolas for MTPC, MTPV and MTPF (with } R_s \neq 0 \text{ & } L_m \neq 0!)\]

Step 3: Intersection of two quadrics (e.g. voltage ellipse and current circle)

\[\mathbf{i}_{s,\text{ref}}^k := \arg \min_{\|\mathbf{i}_s^k\|} \left\{ \mathbf{i}_s^k \in \mathbb{R}^2 \mid Q_A(\mathbf{i}_s^k) = 0 \land Q_B(\mathbf{i}_s^k) = 0 \right\} \]

\[\implies \text{Optimal operation point } \star (\text{reference current})\]

Both lead to subproblem of solving a fourth-order (quartic) polynomial

\[\chi(\lambda) := c_4 \lambda^4 + c_3 \lambda^3 + c_2 \lambda^2 + c_1 \lambda + c_0 = 0\]

\[\implies \text{Analytical solutions exist (e.g. Euler's solution, see [42])}\]
Operation management and operation strategies

Operation management: Operation strategies and transition points

- **Maximum Torque per Current (MTPC):** \( \omega_k \leq \omega_{k, \text{feas}} (m_{m, \text{ref}}) \) (and \( m_{m, \text{ref}} \leq m_{m, \text{nom}} \))
- **Field Weakening (FW):** \( \omega_{k, \text{feas}} (m_{m, \text{ref}}) < \omega_k \) and \( m_{m, \text{ref}} < m_{m, \text{feas}} (\omega_k) \)
- **Maximum Current (MC):** \( \omega_{k, \text{nom}} < \omega_k < \omega_{k, \text{cut-in}} \) and \( m_{m, \text{ref}} \geq m_{m, \text{feas}} (\omega_k) \)
- **Maximum Torque per Voltage (MTPV):** \( \omega_k > \omega_{k, \text{cut-in}} \) and \( m_{m, \text{ref}} \geq m_{m, \text{cut-in}} (\omega_k) \)
Operation management and operation strategies

Maximum Torque per Current (MTPC): Reference torque feasible

Optimization problem

\[
\begin{align*}
\max & -\|i_s^k\|^2 \\
\text{s.t.} & \quad \begin{pmatrix} i_s^k \end{pmatrix}^\top T_i^k + 2 t^\top i_s^k - m_{m,\text{ref}} = 0 \\
& \quad = m_m(i_s^k)
\end{align*}
\]

Solution set

\[
\text{MTPC} := \{ i_s^k \in \mathbb{R}^2 \mid \begin{pmatrix} i_s^k \end{pmatrix}^\top M_C i_s^k + 2 m_C^\top i_s^k = 0 \}
\]

Optimal current reference (★)

\[
i_{s,\text{ref}}^k = \text{MTPC} \cap T(m_{m,\text{ref}})
\]
Operation management and operation strategies

Field Weakening (FW): Reference torque feasible

Optimization problem

\[
\begin{align*}
\text{max} & \quad -\left\| i_s^k \right\|^2 \\
n & \quad i_s^k \in S \\
(i_s^k)^\top T i_s^k + 2 t^\top i_s^k - m_{m,\text{ref}} = 0 \\
& \quad = m_m(i_s^k)
\end{align*}
\]

Feasible set

\[
\boxed{\text{FW}(m_{m,\text{ref}}, \omega, \hat{u}_{\text{max}}) := S(\omega, \hat{u}_{\text{max}}, \hat{i}_{\text{max}}) \cap T(m_{m,\text{ref}})}
\]

Optimal current reference \(\star\)

\[
i_{s,\text{ref}} = \partial \nabla(\omega, \hat{u}_{\text{max}}) \cap T(m_{m,\text{ref}})
\]
Operation management and operation strategies

Maximum Torque per Voltage (MTPV): Reference torque not feasible

**Optimization problem**

\[
\begin{align*}
\text{max} & \quad -\|u_s^k(i_s^k)\|^2 \\
\text{s.t.} & \quad (i_s^k)^\top T i_s^k + 2 t^\top i_s^k - m_{m,\text{ref}} = 0 \\
& \quad = m_m(i_s^k)
\end{align*}
\]

**Solution set**

\[
\begin{align*}
\text{MTPV}(\omega_k) := \{ i_s^k \in \mathbb{R}^2 \mid (i_s^k)^\top M_V i_s^k + 2 m_V^\top i_s^k + \mu_V = 0 \}
\end{align*}
\]

**Optimal current reference (★)**

\[
i_s^k,\text{ref} = \text{MTPV}(\omega_k) \cap \partial V(\omega_k, \hat{u}_{\text{max}})
\]
Operation management and operation strategies

Illustration of the effect of neglecting $R_s$ and/or $L_m$

\[ \text{MTPV}(\omega_k) \quad \text{T}(m_{\text{m,ref}}) \]

- $R_s = 0$ (and $L_m \neq 0$)
- $L_m = 0$ (and $R_s \neq 0$)
- $R_s = 0$ and $L_m = 0$

\[ \Rightarrow \text{Reduced efficiency!} \]

(and wrong torque and/or possibly wrong operation strategies)
Analytical solution

Example: Analytical solution for intersection points of two quadrics

Problem

\[ X := \left\{ x \in \mathbb{R}^2 \mid \begin{array}{l} x^T Ax + 2a^T x + \alpha = 0 \\ x^T Bx + 2b^T x + \beta = 0 \end{array} \right\} =: Q_A(x) \cup Q_B(x) \]

Solution: Relate both (e.g., if \( \alpha \neq 0 \) and \( \beta \neq 0 \))

\[ Q_D(x) := \frac{Q_A(x)}{\alpha} - \frac{Q_B(x)}{\beta} = x^T \left( \frac{A}{\alpha} - \frac{B}{\beta} \right) x + 2 \left( \frac{a}{\alpha} - \frac{b}{\beta} \right)^T x + \left( \frac{\alpha}{\alpha} - \frac{\beta}{\beta} \right) = 0 \]

and note that

\[ Q_D(x) := x^T \left( D x + 2d \right) \overset{!}{=} 0 \iff x^*(\lambda) = -2(D - \lambda J)^{-1} d \]

\( \overset{!}{=} \lambda J x \)
Analytical solution

Example: Analytical solution for intersection points of two quadrics (cont’d)

then insert \( \mathbf{x}^*(\lambda) \) in \( Q_A(\mathbf{x}) \) (or \( Q_B(\mathbf{x}) \)), i.e.

\[
Q_A(\mathbf{x}^*(\lambda)) = 4d^\top (\mathbf{D} - \lambda \mathbf{J})^{-\top} \mathbf{A} (\mathbf{D} - \lambda \mathbf{J})^{-1} d - 4a^\top (\mathbf{D} - \lambda \mathbf{J})^{-1} d + \alpha = 0 \mid \cdot \left[ \det(\mathbf{D} - \lambda \mathbf{J}) \right]^2 \propto \lambda^2
\]

\[\implies \chi(\lambda) := c_4 \lambda^4 + c_3 \lambda^3 + c_2 \lambda^2 + c_1 \lambda + c_0 \triangleq 0\]

\[\implies \text{Analytical (real) solutions } \lambda^* \in \{\lambda_1, \ldots, \lambda_4\} \text{ can be computed [42]:}\]

\[\implies \mathbf{x}^*(\lambda^*) = -2(\mathbf{D} - \lambda^* \mathbf{J})^{-1} d\]
Implementation results

Laboratory setup

Reluctance SM (RSM) and Permanent-magnet SM

Real-time system and VSIs

Host PC (rapid prototyping)
Implementation results

Nonlinear flux linkages and inductances of RSM
Implementation results
Comparison of numerical and analytical solutions for MTPC

Cartesian coordinates

Polar coordinates
Implementation results

Comparison of the computational load (Euler’s solution for fourth-order polynomials)

- Execution times for $N = 10^6$ runs (downsampled by factor 100);
- Average execution times $\mu_n = 43.4 \cdot 10^{-6}$ s and $\mu_a = 7.23 \cdot 10^{-6}$ s; and
- Standard deviations $\sigma_n = 1.61 \cdot 10^{-6}$ s and $\sigma_a = 0.73 \cdot 10^{-6}$ s.
Conclusion

Summary and future work

To take home

- **unified** theory for MTPC, FW, MTPV/MTPF or MC based on
  - **quadrics** (implicit formulation),
  - **optimization problems** with **equality constraint**, and
  - **intersection** of two quadrics

- **analytical** solution for reference currents (& transition points)

- $R_s \neq 0$ and $L_m \neq 0$ (both can be considered **simultaneously**)

- **fast(er) and more accurate** computation

- **applicable** in real-world (nonlinear RSM)

Future work

- convergence (mathematical proof or consideration of nonlinear flux linkages)

- overall implementation, validation and test

- consideration of iron losses (including e.g. $R_{fe}(\omega_k)$)

- impact of parameter uncertainty (e.g. $R_s^a \neq R_s^b \neq R_s^c$)
References


References II


References III


References IV


References V


References VI


References VII
