MPC with analytical solution and integral error feedback for LTI MIMO systems and its application to current control of grid-connected power converters with LCL-filter

Christoph Hackl

Munich School of Engineering (MSE)
Research group “Control of Renewable Energy Systems (CRES)”
Technische Universität München (TUM)

05.10.2015

PRECEDE 2015
Valparaiso, Chile
Model-predictive control: Problem statement

- **Optimality** with respect to performance index
  \[ J = \sum_{i=1}^{n} w_i J_i, \quad n \in \mathbb{N}, \quad \forall i \in \{1, \ldots, n\}: \quad J_i, w_i \geq 0. \]  
  (COST)

- but **not** necessarily “optimal” with respect to sub-cost \( J_i \)
  - How to choose weighting factors \( w_1, \ldots, w_n \)?
    \[ \Rightarrow \text{“Increase } w_i \text{ to “optimize” sub-cost } J_i \text{” does not hold in general [1]!} \]
  - Does (COST) reflect the “human performance index” adequately?
    \[ \Rightarrow \text{Is the use of adaptive weights beneficial [1] ?} \]

- How to achieve “good” control performance under **disturbances** or **uncertainties** (i.e. steady state accuracy and/or no overshoots)?
  - Use of a PI controller (but not admissible) \( \rightsquigarrow \) Integral error feedback
  - Use of long prediction horizons (not feasible) \( \rightsquigarrow \) MPC with analytical solution
Contents

1 Model predictive control: Revisiting theory
   - Considered system class
   - Analytical solution
   - Integral error feedback and system extension
   - Implementation

2 Application
   - Current control of grid-connected power converter with LCL filter
   - Model of grid-connected power converter with LCL filter

3 Simulation results

4 Summary
Model predictive control

Considered system class

LTI MIMO system (discrete) with disturbances

\[
\begin{align*}
  x[k+1] &= Ax[k] + Bu[k] + B_d d[k], \quad x[0] = x_0 \in \mathbb{R}^n \\
  y[k] &= Cx[k],
\end{align*}
\]

(1)

Standard assumption [2, p. 48] on system (1):

(A1) controllability, i.e. \( \text{rank}[B, AB, \ldots, A^{n-1}B] = n \).

(A2) full state feedback, i.e. \( x[k] \) is available for all \( k \)

( or observability, i.e. \( \text{rank}\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} = n \) (observer design).)

(A3) model-based prediction, i.e. \( A, B, B_d \) and \( C \) are known and \( d(\cdot) \) is measured. What if model is not accurate / disturbances are only roughly know?
Model predictive control

Problem & Solution

**Problem:** Find *optimal control action* (vector)

\[ u^H_p[k] := (u^H_p[k + 1]^T, \ldots, u^H_p[k + n_p]^T)^T \in \mathbb{R}^{mn_p} \]  \( \text{(2)} \)

over \( H_p(k) := \{ k + 1, \ldots, k + n_p + 1 \} \) which maximizes *performance index*

\[
J_0(k, u, x, v) = \gamma(k, u, x)^T v + \frac{1}{2} v^T \Lambda(k, u, x) v,
\]

where \( \forall (k, u, x) \in \mathbb{N}_0 \times \mathbb{R} \times \mathbb{R}^n: \)

\[
\begin{align*}
(C_1) & \quad 0 < \Lambda(k, u, x) = \Lambda(k, u, x)^T \in \mathbb{R}^{n_p \times n_p} \quad \text{and} \\
(C_2) & \quad \Lambda(k, u, x)^{-1} \gamma(k, u, x) = 0_{n_p} \iff \gamma(k, u, x) = 0_{n_p}
\end{align*}
\]  \( \text{(3)} \)

**Analytical solution:** *(unique and globally optimal)*

\[ u^H_p[k] := \arg \min_v \left( J_0(k, u[k], x[k], v) \right) = -\Lambda(k, u[k], x[k])^{-1} \gamma(k, u[k], x[k]). \]  \( \text{(4)} \)
**Model predictive control**

Integral error feedback and system extension

Additional state for integration of error

\[
x_i[k + 1] = x_i[k] + T_s k_i \left( \hat{y}_{\text{ref}}[k] - C x[k] \right), \quad k_i > 0,
\]

(5)

Extended system

\[
\begin{pmatrix}
    \bar{x}[k + 1] \\
    \bar{x}_i[k + 1]
\end{pmatrix} =
\begin{pmatrix}
    A & 0 & B_d & O_{n \times m} & d[k] \\
    -T_s k_i C & I_{m \times m} & 0 & O_{n \times m} & \hat{y}_{\text{ref}}[k] \\
    B_d & O_{n \times l} & T_s k_i I_m & 0 & 0
\end{pmatrix}
\begin{pmatrix}
    \bar{x}[k] \\
    T_s k_i \bar{x}_i[k]
\end{pmatrix}
\]

\[
\bar{y}[k] = C k_c I_m \bar{x}[k], \quad k_c > 0.
\]
Model predictive control
Implementation of MPC with analytical solution (AS) and integral error feedback (IEF)

\[ \bar{x} = (x) \quad \Rightarrow \quad \bar{d} = (d'_{ref}) \]

\( \hat{y}_{H_p}^{ref} \)
\( \hat{d}_{H_p}^{ref} \)
\( u \)
\( \bar{E} \)
\( \bar{F} \)
\( F_{d} \)
\( G \)
\( G_{d} \)
\( W_{e} \)
\( W_{u} \)

MPC with AS and IEF

**Performance index (with tracking and control weights):**

\[
J(k, u, \bar{x}, u_{*}^{H_p}) = (\hat{e}_{H_p}^{H_p})^T W_{e}(k, u, \bar{x}) \hat{e}_{H_p}^{H_p} + (u_{*}^{H_p})^T W_{u}(k, u, \bar{x}) u_{*}^{H_p}
\]

\[
=: J_{e}(k, u, \bar{x}, u_{*}^{H_p})
\]

\[
=: J_{u}(u_{*}^{H_p})
\]

with

\[
\hat{e}_{H_p}^{H_p}[k] = \hat{y}_{ref}^{H_p}[k] - (\bar{E}[x][k] + \bar{F}u[k] + \bar{F}_{d}[d][k] + \bar{G}_{d}[\hat{d}_{H_p}^{H_p}[k] + \bar{G}u_{*}^{H_p}[k])
\]

\[=: \tilde{y}[k] \]

where

\[
\bar{E} := \begin{bmatrix} \frac{CA}{CA^2} \\ \vdots \\ \frac{CA^{n_p+1}}{CA^{n_p+1}} \end{bmatrix}, \bar{F} := \begin{bmatrix} \frac{CB}{CAB} \\ \vdots \\ \frac{CAB_{d}}{CAB_{d}} \end{bmatrix}, \bar{F}_{d} := \begin{bmatrix} \frac{CB_{d}}{CAB_{d}} \\ \vdots \\ \frac{CB_{d}}{CAB_{d}} \end{bmatrix}, \bar{G}(\bar{C}, A, B), \bar{G}_{d}(\bar{C}, A, B_{d})
\]

**Performance index (reformulated):**

\[
J_{0}(k, u, \bar{x}, u_{*}^{H_p}) = \gamma(k, u, \bar{x})^T u_{*}^{H_p} + \frac{1}{2} (u_{*}^{H_p})^T \Lambda(k, u, \bar{x}) u_{*}^{H_p}
\]

where

\[
\gamma(k, u, \bar{x}) = 2(-\hat{y}_{ref}^{H_p}[k]^T + \bar{x}^T \bar{E}^T + u^T \bar{F}^T + d[k]^T \bar{F}_{d}^T + \hat{d}_{H_p}^{H_p}[k]^T \bar{G}_{d}^T) W_{e}(k, u, \bar{x}) \bar{G}.
\]

and

\[
\Lambda(k, u[k], \bar{x}[k]) := 2 \left( \bar{G}^T W_{e}(k, u[k], \bar{x}[k]) \bar{G} + W_{u}(k, u[k], \bar{x}[k]) \right)
\]

\[
\Rightarrow u_{*}^{H_p}[k] = \Lambda(k, u[k], \bar{x}[k])^{-1} \gamma(k, u[k], \bar{x}[k]).
\]

\[\Rightarrow (Almost) arbitrary lengths n_p \gg 1 of prediction horizon feasible!\]
Application
Current control of grid-connected power converter with LCL filter

Assumption: Full state feedback available (otherwise observer design needed [3]).
Application
Model of grid-connected power converter with LCL filter

\[ x[k + 1] = \begin{pmatrix} A \\ B \\ B_d \end{pmatrix} x[k] + \begin{pmatrix} I_6 + T_s A_c \\ T_s B_c \\ T_s B_{c,d} \end{pmatrix} u[k] + \begin{pmatrix} C_c \\ \end{pmatrix} d[k] \]

- system is controllable [3]
- full state feedback available (system is observable [3])
- grid voltage is measured \( d = k u_{g}^k \), non-ideal measurement assumed
Simulation results \( (k_u = 0.85) \)

\((d, q)\)-signals: \( \text{MPC, MPC with IEF \& MPC with IEF and } y^H_{ref} (n_p = 25) \)

\[
\hat{u}_g \ [V] \\
\hat{i}_d \ [A] \\
\hat{i}_q \ [A] \\
\|u_f^k\| \ [V] \\
\text{time } t \ [s]
\]

\[0 \ 0.1 \ 0.2 \ 0.3 \ 0.4 \ 0.5\]

\[0 \ 250 \ 500 \]

- \(\hat{u}_g\) = 100, 150, 200
- \(\hat{i}_d\) = -20, 0, 20
- \(\hat{i}_q\) = -10, 0, -20
- \(|u_f^k|\) = 250, 275, 300

\(05.10.2015\)

“MPC with analytical solution \& integral error feedback for VSI with LCL-filter”, C. Hackl

10/13
Simulation results \( (k_u = 0.85) \)

Phase \( a \) signals:  
- **MPC**,  
- **MPC with IEF &**  
- **MPC with IEF and**  

\[ y_{	ext{ref}}^{H_p} (n_p = 25) \]
Summary
To take home and open questions

- MPC with integral error feedback
  - *fits theory* (extended system with additional state(s) is still a LTI MIMO system) and
  - *is beneficial* (compensates for disturbances and uncertainties)!

- Long prediction horizons
  - *are beneficial* (faster tracking performance; e.g. without overshoots) and
  - *are feasible with analytical solution* (computational load does not increase exponentially)!

- Exploitation of non-constant weights $W_e(k, u, \bar{x})$ and $W_u(k, u, \bar{x})$ feasible? E.g. to incorporate soft-constraints for
  - saturated control actions [1], i.e. $w_u \propto \frac{1}{u_{\text{max}} - \|u[k]\|}$ or
  - prescribed transient accuracy, i.e. $w_e \propto \frac{1}{\psi[k] - \|e[k]\|}$ [1, 4]?

- Apply results to nonlinear systems (with online linearization)?
References


Simulation results

Implementation data

<table>
<thead>
<tr>
<th>description</th>
<th>symbols &amp; values</th>
</tr>
</thead>
<tbody>
<tr>
<td>sampling time</td>
<td>$T_s = 1.25 \cdot 10^{-4}$ s</td>
</tr>
<tr>
<td>LCL-filter</td>
<td>$L_f = 6.25 \cdot 10^{-3}$ H, $L_g = 26.4 \cdot 10^{-4}$ H, $C = 4.95 \cdot 10^{-4}$ F, $R = R_f = R_g = 0$ Ω</td>
</tr>
<tr>
<td>VSI</td>
<td>$f_{pwm} = 8$ kHz, $u_{dc} = 1000$ V</td>
</tr>
<tr>
<td>PLL</td>
<td>$k_u = 0.85$ (15 % grid amplitude error)</td>
</tr>
<tr>
<td>MPC with AS</td>
<td>$n_p = 25$, $W_u = 5000 \cdot I_{54}$ 1/A, $W_e = 50 \cdot I_{50}$ 1/V. $k_i = 50$, $k_c = 2$ (integral error feedback)</td>
</tr>
</tbody>
</table>

Table: System, implementation and controller data.