Path-following funnel control for rigid-link revolute-joint robotic systems

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Outline

1. Problem statement
2. Motivation for high-gain adaptive position control
3. Position funnel control for robots
4. Path-following funnel control for robots
5. Example and simulation results
6. Conclusion
Position control of industrial robots

Applications

- (spot-)welding
- painting/enameling
- mounting/assembling
- laser-beam cutting
- ...

Goals:

- precise position control (of end effector), i.e. for given $\lambda > 0$:

  \[
  \forall t \geq t_0: \quad \|e_E(t)\| = \|y_{ref,E}(t) - y_E(t)\| \leq \lambda.
  \]

- controller: simple and easy to tune
Rigid robotic manipulator with $n$ revolute joints

Robot model (in joint space)

\[
\begin{align*}
M(y(t))\ddot{y}(t) + C(y(t), \dot{y}(t))\dot{y}(t) + (\mathcal{F}\dot{y})(t) + g(y(t)) + d(t) &= u(t), \\
(y(0), \dot{y}(0))^T &= (y_0, y_1)^T \in \mathbb{R}^{2n}
\end{align*}
\]

(RO)

- **standard assumptions**
  - $\exists \bar{c}_M, c_M > 0 \ \forall y \in \mathbb{R}^n : \ 0 < c_M I_n \leq M(y) = M(y)^T \leq \bar{c}_M I_n$
  - $\exists c_C > 0 \ \forall y, v \in \mathbb{R}^n : \ ||C(y, v)v|| \leq c_C ||v||^2$
  - $\mathcal{F} : C(\mathbb{R}_{\geq 0}; \mathbb{R}) \rightarrow \mathcal{L}^\infty_{loc}(\mathbb{R}_{\geq 0}; \mathbb{R})$ (causal friction operator)
  - $\exists c_g > 0 \ \forall y \in \mathbb{R}^n : \ \|g(y)\| \leq c_g$
  - $d(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0}; \mathbb{R}^n)$ (disturbance)
  - $u(\cdot) \in C(\mathbb{R}_{\geq 0}; \mathbb{R}^n)$ (control input)
  - Feedback signals: $y(\cdot)$ and $\dot{y}(\cdot)$

- ‘structural properties’
  - (strict) relative degree $r = 2$
  - Positive (definite) high-frequency gain
  - No internal dynamics (minimum-phase)
  - Derivative feedback admissible
High-gain adaptive control

Promising advantages

- ‘structural system knowledge’ sufficient (robustness)
  - known relative degree \( r \geq 1 \)
  - positive high-frequency gain (or known sign)
  - minimum-phase
- high feedback gains reduce ‘stick-slip’
- reference tracking with
  - ‘prescribed asymptotic accuracy’ (see e.g. [1, 2]) and
  - ‘prescribed transient accuracy’ (see e.g. [3–5])

Adaptive \( \lambda \)-tracking control

Funnel control
Relative degree two, high gains & derivative feedback

controller

\[-k(y + k_D \dot{y})\]

system

\[u \quad F_2(s) = \frac{(s+5)}{(s-1)^2(s+1)} \quad y \]

\[\dot{y} \]

‘Structural properties’ of \(F_2(s)\)

- relative degree (pole excess):
  \(r = 2\)
- positive high-frequency gain
  \(\lim_{s \to \infty} s^2 F_2(s) = 1\)
- minimum-phase
  (Hurwitz numerator)

\[\exists k^* > 0, \text{ such that closed-loop is stable for } k_D > 0 \text{ and all } k > k^*\]
Control objective

- reference tracking with ‘prescribed transient accuracy’ for each joint $i \in \{1, \ldots, n\}$:

$$\forall t \geq 0: |e_i(t)| = |y_{\text{ref},i}(t) - y_i(t)| < \psi_{0,i}(t) \quad \text{and} \quad |\dot{e}_i(t)| < \psi_{1,i}(t)$$

- joint reference $y_{\text{ref},i}(\cdot) \in C^1(\mathbb{R}_{\geq 0}; \mathbb{R})$, $y_{\text{ref},i}(\cdot), \dot{y}_{\text{ref},i}(\cdot), \ddot{y}_{\text{ref},i}(\cdot) \in L^\infty(\mathbb{R}_{\geq 0}; \mathbb{R})$

- joint funnel boundary $(\psi_{0,i}(\cdot), \psi_{1,i}(\cdot)) \in C^1(\mathbb{R}_{\geq 0}; [\lambda, \infty)^2)$, $\lambda > 0$
MIMO funnel controller (extension of [6, 7])

\[ u(t) = M(y(t)) \left( K_0(t)^2 e(t) + K_0(t) K_1(t) \dot{e}(t) \right) \]

- \( e(t) = (e_1(t), \ldots, e_n(t))^\top = (y_{ref,1}(t) - y_1(t), \ldots, y_{ref,n}(t) - y_n(t))^\top \)
- \( K_0(t) = \text{diag} \{ k_{0,1}(t), \ldots, k_{0,n}(t) \} \) & \( K_1(t) = \text{diag} \{ k_{1,1}(t), \ldots, k_{1,n}(t) \} \)
- \( \forall i \in \{1, \ldots, n\}: \quad k_{0,i}(t) = \frac{1}{\psi_{0,i}(t) - |e_i(t)|} \) and \( k_{1,i}(t) = \frac{1}{\psi_{1,i}(t) - |\dot{e}_i(t)|} \)
Properties of closed-loop system [8, Theorem 3.1]

\( (\text{RO}): M(y(t)) \ddot{y}(t) + C(y(t), \dot{y}(t)) \dot{y}(t) + (\mathcal{S}\dot{y})(t) + g(y(t)) + d(t) = u(t) \)

\( (\text{FC}_n): \ u(t) = M(y(t)) \left( K_0(t)^2 e(t) + K_0(t) K_1(t) \dot{e}(t) \right) \)

For **known** \( M(y) \):

- \( n_m(\cdot) \in \mathcal{W}^{2, \infty}(\mathbb{R}_{\geq 0}; \mathbb{R}^n) \) (measurement errors/noise admissible)
- \( \forall i \in \{1, \ldots, n\} \ \exists \varepsilon_i > 0 \ \forall t \geq 0: \ \psi_{0,i}(t) - |e_i(t)| \geq \varepsilon_i \) and \( \psi_{1,i}(t) - |\dot{e}_i(t)| \geq \varepsilon_i \)
- \( K_0(\cdot), K_1(\cdot) \in \mathcal{L}^{\infty}(\mathbb{R}_{\geq 0}; \mathbb{R}^n) \)
- \( u(\cdot) \in \mathcal{L}^{\infty}(\mathbb{R}_{\geq 0}; \mathbb{R}^n) \)
Control objective

- **path-following** with ‘prescribed transient accuracy’ for each joint \( i \in \{1, \ldots, n\} \):

\[
\forall t \geq 0: \quad |e_i(t)| = |p_i(\theta(t)) - y_i(t)| < \psi_{0,i}(t) \quad \text{and} \quad |\dot{e}_i(t)| < \psi_{1,i}(t)
\]

- path: \( p(\cdot) \in C^2(\mathbb{R}; \mathbb{R}^n) \) and \( p(\cdot), \frac{d}{dt} p(\cdot), \frac{d^2}{dt^2} p(\cdot) \in L^\infty(\mathbb{R}; \mathbb{R}^n) \)

- timing dynamics with virtual input \( v \):

\[
\frac{d^2}{dt^2} \theta(t) = v(t), \quad (\theta(0), \dot{\theta}(0))^\top = (\theta_0, \theta_1)^\top \in \mathbb{R}^2 \quad \text{(TD)}
\]
MIMO path-following funnel controller

- funnel controller

\[ u(t) = M(y(t)) \left( K_0(t)^2 e(t) + K_0(t) K_1(t) \dot{e}(t) \right) \]  

(FC\(_n\))

- timing feedback (virtual input \( v \))

\[ v(t) = -\kappa(e(t), \dot{e}(t)) \begin{bmatrix} \theta(t) \\ \dot{\theta}(t) \end{bmatrix}, \quad \kappa(\cdot, \cdot) = \begin{pmatrix} \kappa_1(\cdot, \cdot) \\ \kappa_2(\cdot, \cdot) \end{pmatrix} \in C^1(\mathbb{R}^n \times \mathbb{R}^n; \mathbb{R}^2) \]  

(TF)
Properties of closed-loop system [Theorem 2]

- (RO): 
  \[ M(y(t)) \ddot{y}(t) + C(y(t), \dot{y}(t)) \dot{y}(t) + (\mathcal{S}\dot{y})(t) + g(y(t)) + d(t) = u(t) \]

- (FC\(_n\)): 
  \[ u(t) = M(y(t)) \left( K_0(t)^2 e(t) + K_0(t) K_1(t) \dot{e}(t) \right) \]

- (TD), (TF): 
  \[ \ddot{\theta}(t) = -\kappa(e(t), \dot{e}(t))^\top \left( \begin{array}{c} \dot{\theta}(t) \\ \dot{\theta}(t) \end{array} \right), \kappa(\cdot, \cdot) \in C^1(\mathbb{R}^n \times \mathbb{R}^n; \mathbb{R}^2) \]

- For known \( M(y) \) and \( \exists 0 < \underline{\kappa} \leq \bar{\kappa}: \underline{\kappa} \leq \kappa_1(\alpha, \beta) \leq \kappa_2(\alpha, \beta) \leq \bar{\kappa} \) and \( 1 + \underline{\kappa} \leq \kappa_2(\alpha, \beta) \) for all \( \alpha, \beta \in \mathbb{R}^n \) then
  - \( n_m(\cdot) \in \mathcal{W}^2,\infty(\mathbb{R}_{\geq 0}; \mathbb{R}^n) \) (measurement errors admissible)
  - \( \forall i \in \{1, \ldots, n\} \exists \varepsilon_i > 0 \forall t \geq 0: \psi_{0,i}(t) - |e_i(t)| \geq \varepsilon_i \) and \( \psi_{1,i}(t) - |\dot{e}_i(t)| \geq \varepsilon_i \)
  - \( K_0(\cdot), K_1(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0}; \mathbb{R}^n_{\geq 0}) \) and \( u(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0}; \mathbb{R}^n) \)
  - \( \lim_{t \to \infty} \theta(t) = 0 \) (timing dynamics are asymptotically stable [9, Thm. 2.15])
Example: Planar robot and path

\[ u(t) = M(y(t)) \begin{pmatrix} k_{0,1}(t)^2 & 0 \\ 0 & k_{0,2}(t)^2 \end{pmatrix} e(t) + \begin{pmatrix} k_{0,1}(t)k_{1,1}(t) & 0 \\ 0 & k_{0,2}(t)k_{1,2}(t) \end{pmatrix} \dot{e}(t) \]

\[ v(t) = -\alpha(e(t), \dot{e}(t))\kappa^T \begin{pmatrix} \theta(t) \\ \dot{\theta}(t) \end{pmatrix} \]

with \( \kappa = (10, 100)^T \) and

\[ 0 < \alpha_{\min} \leq \alpha(e, \dot{e}) = \frac{\alpha_{\max} + \alpha_{\min}(\|e\| + \|\dot{e}\|)}{\|e\| + \|\dot{e}\| + 1} \leq \alpha_{\max} \]
Simulation results: Errors, control & timing dynamics

\begin{align*}
\dot{e}_1(t) & = \dot{e}_2(t) = \psi_0(t) \\
\dot{\theta}(t) & = \frac{\dot{e}_1(t)}{\dot{e}_2(t)} = \frac{\psi_1(t)}{\psi_2(t)} \\
\alpha(t) & = \frac{u_1(t)}{u_2(t)} = \frac{d_1(t)}{d_2(t)}
\end{align*}

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“Path-following funnel control for rigid-link revolute-joint robotic systems”, C. Hackl
Simulation results: Path and trajectory
Summary and future work

To take home:

- position funnel control: simple but time-varying PD-controller
- measurement noise admissible
- no compensation of
  - friction
  - disturbances (unknown but bounded)
  - Coriolis and/or gravity effects
- however: inertia matrix must be known
- path-following funnel control with prescribed path accuracy
- timing dynamics with virtual input (one degree of freedom more)

Open questions

- derivation of feasibility condition (actuator saturation)?
- utilization of timing dynamics to assure feasible control actions?
- parametrization of funnel boundary by timing variables?
- path-following in finite time?
References


