Funnel-Control with Online Foresight

Christoph M. Hackl and Dierk Schröder
Overview

1. Funnel-Control (FC)
   - Prerequisites, Basic Idea -

2. Scaling Function

3. Online Foresight: Minimal (Future) Distance (MD)

4. Simulation

5. Conclusion & Outlook
Funnel-Control (FC) – Prerequisites
(developed by Ilchmann et al.)

Controller properties:
- Proportional Controller
- Adaptive Time-Varying Gain
- Control Law:
  \[ u(t) = \alpha \left( \partial \phi(t), \Psi(t), \|e(t)\| \right) \cdot e(t) \]

System of Class \( S \)- prerequisites:
- Relative Degree \( r = 1 \)
- Minimum-Phase (stable zero-dynamics)
- Known High-Frequency Gain

**Fig. 1: Block diagramm – System of Class S controlled with FC**
Funnel-Control (FC) with Vertical Distance (VD) - Basic Idea

Adaption of gain depending on (vertical) distance:

\[ \alpha_D (t) = \frac{1}{d_V (t)} \]

- Short distance => high gain (more aggressive control)
- Great distance => small gain (more relaxed control)

Time-varying (adaptive) gain \( \alpha(\square) \) \( \Rightarrow \) Boundedness of control error (for sufficiently large but finite control inputs!)

\[
\alpha \left( \partial \mathcal{F}_\phi (t), \Psi (t), \| e(t) \| \right) = \Psi (t) \cdot \alpha_D (t) = \frac{\Psi (t)}{d_V (t)} = \frac{\Psi (t)}{\partial \mathcal{F}_\phi (t) - \| e(t) \|} \quad 0 \leq \partial \mathcal{F}_\phi (t)
\]
Scaling Function

- any continuous, bounded and positive function is allowed: \( \Psi(t) > 0 \) for all \( t \geq t_0 \)

- to choose e.g. minimal gain ("memory effect"): 
  \[
  \alpha(t) = \frac{\Psi(t)}{\partial F_\varphi(t) - \|e(t)\|} \geq \frac{\Psi(t)}{\partial F_\varphi(t)}
  \]

- **only to increase** the distance gain (acceleration effect):

\[
\alpha(t) = \Psi(t) \cdot \alpha_D(t) \geq \alpha_D(t)
\]

\[
\Rightarrow \Psi(t) \geq 1 \text{ for all } t \geq t_0
\]

(otherwise the principle of Funnel Control may be deteriorated)

- if Funnel Boundary itself is used for scaling:

\[
\Psi(t) = \partial F_\varphi(t) + (1 - \mu) \text{ if } \lim_{t \to \infty} \partial F_\varphi(t) = \mu < 1
\]
Online Foresight: Minimal (Future) Distance (MD)

![Graph](image)

**Fig. 3: Vertical (VD) & Minimal (Future) Distance (MD)**

- Minimal (Future) Distance (MD) exists with \( t_F \geq t \) and:

\[
d_F(t, t_F) = \min_{t_F \geq t} \sqrt{(\partial F_\varphi(t_F) - \|e(t)\|^2 + (t_F - t)^2} \leq d_V(t)
\]

- Funnel Control more effective with (especially for initial error \( e(t_0) \)) as:

\[
u_F(t) = \alpha_F(t) \cdot e(t) \geq \alpha_V(t) \cdot e(t) = u_V(t)
\]
Simulation Results (Analytic Approach)

Example System:

\[ F_{PT_1}(s) = \frac{2}{1+ss^2} \]

Funnel Boundary and Scaling Function:

\[
\partial \mathcal{F}_A(t) = \begin{cases} 
- \frac{\varphi_0 - \varphi_\infty}{\tau_\mu} \cdot t + \varphi_0 & t < \tau_\mu \\
\varphi_\infty & t \geq \tau_\mu 
\end{cases}
\]

\[
\Psi(t) = \partial \mathcal{F}_A(t) + (1 - \varphi_\infty) \text{ for VD}
\]
Simulation Results (Analytic Approach)

**Example System:**

\[ F_{PT_i}(s) = \frac{2}{1 + s^2} \]

**Funnel Boundary and Scaling Function:**

\[
\partial F_A(t) = \begin{cases} 
- \frac{\varphi_0 - \varphi_{\infty}}{\tau_{\mu}} \cdot t + \varphi_0 & t < \tau_{\mu} \\
\varphi_{\infty} & t \geq \tau_{\mu}
\end{cases}
\]

\[ \Psi(t) = \partial F_A(t) + (1 - \varphi_{\infty}) \text{ for VD} \]
Simulation Results with Noise (Analytic Approach)

Fig. 8a: System Output $y(t)$

Example System:

$$F_{PT_i}(s) = \frac{2}{1 + s^2}$$

Fig. 8b: Error $e(t)$, Funnel Boundary $pF(t)$

Funnel Boundary and Scaling Function:

$$\partial F_A(t) = \begin{cases} 
- \frac{\varphi_0 - \varphi_\infty}{\tau_\mu} \cdot t + \varphi_0 & t < \tau_\mu \\
\varphi_\infty & t \geq \tau_\mu
\end{cases}$$

$$\Psi(t) = \partial F_A(t) + (1 - \varphi_\infty) \text{ for VD}$$
Simulation Results with Noise (Numeric & Derivative Approach)

Fig. 9a: System Output $y(t)$

Example System:

$$F_{PTi}(s) = \frac{2}{1 + s^2}$$

Fig. 9b: Error $e(t)$, Funnel Boundary $pF(t)$

Funnel Boundary and Scaling Function:

$$\partial F_{exp}(t) = \varphi_{exp,0} \cdot \exp(-\frac{t}{T_{exp}}) + \varphi_{exp,\infty}$$

$$\Psi(t) = \partial F_{exp}(t) + (1 - \varphi_{exp,\infty}) \text{ for VD}$$
Simulation Results with Noise (Numeric & Derivative Approach)

Example System:

\[ F_{PT_i}(s) = \frac{2}{1 + s^2} \]

Funnel Boundary and Scaling Function:

\[ \partial \mathcal{F}_{\text{exp}}(t) = \varphi_{\text{exp,0}} \cdot \exp\left(-\frac{t}{T_{\text{exp}}}\right) + \varphi_{\text{exp,}\infty} \]

\[ \Psi(t) = \partial \mathcal{F}_{\text{exp}}(t) + (1 - \varphi_{\text{exp,}\infty}) \text{ for VD} \]
Conclusion:

- Simple, robust and adaptive control structure
- Pre-definition of desired transient behavior possible
- Measurement noise is admissible
- **Improved performance** (Acceleration) for:
  1. *well chosen scaling function*
  2. *implemented minimal (future) distance*
     
     *(derivative method shows best performance with simple realization and free choice of adequate Funnel Boundary)*

Outlook:

- Consideration of constrained (saturated) control inputs
- Active Damping
- Further experimental results in laboratory
Funnel-Control – Achieved Goals

• Extension of the usable system class (reduction of the relative degree)

• Robustness analysis (parameter variations)

• Steady state accuracy (vanishing control error)

Thank you for your attention