Enhanced Funnel-Control with Improved Performance

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Overview

1. Funnel-Control (FC)  
   - Prerequisites, Basic Idea -

2. Scaling Function

3. Online Foresight: Minimal (Future) Distance (MD)

4. Active Damping

5. Simulation

6. Conclusion & Outlook
Funnel-Control (FC) – Prerequisites
(developed by Ilchmann et al.)

Controller properties:
• Proportional Controller
• Adaptive Time-Varying Gain
• Control Law:

\[ u(t) = \alpha \left( \partial \mathcal{F}_\varphi (t), \Psi (t), \lVert e(t) \rVert \right) \cdot e(t) \]

System of Class \( S \) - prerequisites:
• Relative Degree \( r = 1 \)
• Minimum-Phase
  (stable zero-dynamics)
• Known High-Frequency Gain

Fig. 1: Block diagramm – System of Class \( S \) controlled with FC
Funnel-Control (FC) with Vertical Distance (VD) - Basic Idea

Adaption of gain depending on (vertical) distance:

\[
\alpha_D(t) = \frac{1}{d_V(t)}
\]

- Short distance => high gain (more aggressive control)
- Great distance => small gain (more relaxed control)

Time-varying (adaptive) gain \( \alpha(\square) \) \( \Rightarrow \) Boundedness of control error (for sufficiently large but finite control inputs!)

\[
\alpha\left(\partial F_\phi(t), \Psi(t), \|e(t)\|\right) = \Psi(t) \cdot \alpha_D(t) = \frac{\Psi(t)}{d_V(t)} = \frac{\Psi(t)}{\partial F_\phi(t) - \|e(t)\|}
\]

Fig. 2: Error evolution within Funnel Boundary
Scaling Function

• any continuous, bounded and positive function is allowed: \( \Psi(t) > 0 \) for all \( t \geq t_0 \)

• to choose e.g. minimal gain ("memory effect"): 
  \[
  \alpha(t) = \frac{\Psi(t)}{\partial F_\phi(t) - \|e(t)\|} \geq \frac{\Psi(t)}{\partial F_\phi(t)}
  \]
  \( 0 < \|e(t)\| \leq \partial F_\phi(t) \)

• **only to increase** the distance gain (acceleration effect):

  \[
  \alpha(t) = \Psi(t) \cdot \alpha_D(t) \geq \alpha_D(t)
  \]
  \[
  \Rightarrow \quad \Psi(t) \geq 1 \quad \text{for all} \quad t \geq t_0
  \]

(otherwise the principle of Funnel Control may be deteriotated)

• if Funnel Boundary itself is used for scaling:

  \[
  \Psi(t) = \partial F_\phi(t) + (1 - \mu) \quad \text{if} \quad \lim_{t \to \infty} \partial F_\phi(t) = \mu < 1
  \]

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Online Foresight: Minimal (Future) Distance (MD)

- Minimal (Future) Distance (MD) exists with $t_F \geq t$ and:

$$d_F(t,t_F) = \min_{t_F \geq t} \sqrt{(\partial F_\phi(t_F) - \|e(t)\|^2 + (t_F - t)^2 \leq d_V(t)}$$

- Funnel Control more effective with (especially for initial error $e(t_0)$) as:

$$u_F(t) = \alpha_F(t) \cdot e(t) \geq \alpha_V(t) \cdot e(t) = u_V(t)$$
Active Damping: Asymmetric Funnel Boundaries (Intuitive Approach)

- Asymmetric choice of the Funnel Boundary with maximal overshoot limit $\ddot{u}>0$:

$$
\mathcal{F}_{AD} = \left\{ e(t) \left| \begin{array}{l}
-\ddot{u} \cdot y^*(t) < e(t) < \partial \mathcal{F}_\varphi(t) \quad \forall y^*(t) \geq 0 \\
-\partial \mathcal{F}_\varphi(t) < e(t) < -\ddot{u} \cdot y^*(t) \quad \forall y^*(t) < 0
\end{array} \right. \right\}
$$

- Generation of control input for asymmetric boundaries:

$$
u(t) = \begin{cases} 
\Psi(t) \cdot \alpha^+_D(t) \cdot e(t) & , e(t) > 0 \\
\Psi(t) \cdot \alpha^-_D(t) \cdot e(t) & , e(t) \leq 0
\end{cases}
$$

Fig. 6: Asymmetric Funnel Boundary
Simulation Results (Analytic Approach)

**Fig. 7a: System Output y(t)**

**Example System:**

\[
F_{PT_i}(s) = \frac{2}{1 + s^2}
\]

**Funnel Boundary and Scaling Function:**

\[
\begin{align*}
\partial F_A(t) &= \begin{cases} 
- \frac{\varphi_0 - \varphi_\infty}{\tau_\mu} \cdot t + \varphi_0 & t < \tau_\mu \\
\varphi_\infty & t \geq \tau_\mu
\end{cases} \\
\Psi(t) &= \partial F_A(t) + (1 - \varphi_\infty) \text{ for VD}
\end{align*}
\]

**Fig. 7b: Error e(t), Funnel Boundary pF(t)**
**Simulation Results (Analytic Approach)**

![Graph showing Control Output u(t)](image)

*Fig. 7c: Control Output u(t)*

**Example System:**

\[
F_{PT_i}(s) = \frac{2}{1 + s^2}
\]

**Funnel Boundary and Scaling Function:**

\[
\partial F_A(t) = \begin{cases} 
- \frac{\varphi_0 - \varphi_\infty}{\tau_\mu} \cdot t + \varphi_0 & t < \tau_\mu \\
\varphi_\infty & t \geq \tau_\mu
\end{cases}
\]

\[
\Psi(t) = \partial F_A(t) + (1 - \varphi_\infty) \text{ for VD}
\]
Simulation Results with Noise (Analytic Approach)

Fig. 8a: System Output $y(t)$

Example System:

$$F_{PT_i}(s) = \frac{2}{1 + s^2}$$

Fig. 8b: Error $e(t)$, Funnel Boundary $pF(t)$

Funnel Boundary and Scaling Function:

$$\Psi(t) = \partial F_A(t) + (1 - \varphi) \text{ for VD}$$
**Simulation Results with Noise (Numeric & Derivative Approach)**

**Fig. 9a:** System Output $y(t)$

**Example System:**

$$F_{PT_i}(s) = \frac{2}{1 + s^2}$$

**Fig. 9b:** Error $e(t)$, Funnel Boundary $pF(t)$

**Funnel Boundary and Scaling Function:**

$$\partial F_{\exp}(t) = \phi_{\exp,0} \cdot \exp\left(-\frac{t}{T_{\exp}}\right) + \phi_{\exp,\infty}$$

$$\Psi(t) = \partial F_{\exp}(t) + (1 - \phi_{\exp,\infty}) \text{ for VD}$$
Simulation Results with Noise (Numeric & Derivative Approach)

![Control Output u(t)](image)

**Fig. 9c: Control Output u(t)**

Example System:

\[
F_{PT_i}(s) = \frac{2}{1 + s^2}
\]

Funnel Boundary and Scaling Function:

\[
\partial \mathcal{F}_{\exp}(t) = \varphi_{\exp, 0} \cdot \exp\left(-\frac{t}{T_{\exp}}\right) + \varphi_{\exp, \infty}
\]

\[
\Psi(t) = \partial \mathcal{F}_{\exp}(t) + (1 - \varphi_{\exp, \infty}) \text{ for VD}
\]
Simulation Results with Noise (Active Damping)

**Fig. 10a: System Output y(t)**

**Example System:**

\[
F_{PD_{T_3}}(s) = \frac{2(1 + s + s^2 0.25)}{(1 + s 2)(1 + s 0.033 + s^2 0.028)}
\]

**Funnel Boundary and Scaling Function:**

\[
\partial F_{AD+}(t) = \varphi_{exp,0} \cdot \exp(-\frac{t}{T_{exp}}) + \varphi_{exp,\infty}
\]

\[
\partial F_{AD-}(t) = -\dot{u} \cdot y^*
\]
Conclusion:

- Simple, robust and adaptive control structure
- Pre-definition of desired transient behavior possible
- Measurement noise is admissible
- **Improved performance** for:
  1. *well chosen scaling function* (Acceleration)
  2. *implemented minimal (future) distance* (Acceleration)
     *(derivative method shows best performance with simple realization and free choice of adequate Funnel Boundary)*
  3. *asymmetric Funnel boundaries* *(Active Damping)*

Outlook:

- Consideration of constrained (saturated) control inputs *(MED’07: T01-015)*
- Further experimental results in laboratory *(e.g. Flexible Three-Mass-System or Robot Manipulator Control)*
Funnel-Control – Achieved Goals

- Extension of the usable system class (reduction of the relative degree)

- Robustness analysis (parameter variations)

- Steady state accuracy (vanishing control error)

- Saturated Control Input Compensation

Thank you for your attention