Funnel control in mechatronics: An introduction

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Outline

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   - High-gain control

2 Funnel control
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   - Relative-degree-two systems
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   - Speed control (relative-degree-one case)
   - Position control (relative-degree-two case)

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Reference tracking under load (CNC turning machine)

- Problem statement
  - reference \((\omega_{\text{ref}}, x_{\text{ref}}, y_{\text{ref}}) : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^3\)
  - precise position and speed control, e.g. for prescribed \(\lambda > 0\)

\[
\forall t \geq t_0 : |e(t)| = |y_{\text{ref}}(t) - y(t)| \leq \lambda.
\]

- Challenges
  - nonlinear effects (e.g. actuator saturation, friction)
  - friction and loads (disturbances) unknown and varying
  - system parameters (solely) roughly known
High-gain control - an intuition

controller

\[-k(y + k_D \dot{y})\]

system

\[F_2(s) = \frac{(s+5)}{(s-1)^2(s+1)}\]

‘structural properties’ of \(F_2(s)\)

- relative degree (pole excess):
  \(r = 2\)

- positive high-frequency gain
  \(\lim_{s \to \infty} s^2 F_1(s) = 1\)

- minimum-phase
  (numerator is Hurwitz)

Root-locus (\(\times\) poles, \(\circ\) zeros)

\[k_D = 0 \quad k_D = 1/4\]
Control objective and funnel controller \((r = 1)\)

- **objective:** ‘tracking with prescribed transient accuracy’ of reference \(y_{\text{ref}}(\cdot)\)

- **funnel controller:**

\[
\begin{align*}
\psi_0(0) &\quad \psi_0(t) \\
e(0) &\quad e(t) \\
-\lambda_0 &\quad \psi_0(\cdot) \\
-\psi_0(0) &\quad \psi_0(t)
\end{align*}
\]

where

\[
\begin{align*}
e(t) &= y_{\text{ref}}(t) - y(t), \\
y_{\text{ref}}(\cdot) &\in \mathcal{C}(\mathbb{R}_{\geq 0}; \mathbb{R}), \\
\psi_0(\cdot) &\in \mathcal{C}(\mathbb{R}_{\geq 0}; [\lambda_0, \infty))
\end{align*}
\]

with \(\lambda_0 > 0\)

- **funnel controller:**

\[
u(t) = k_0(t)e(t) \quad \text{where} \quad k_0(t) = \frac{s_0(t)}{\psi_0(t) - |e(t)|}
\]

\((\text{FC}_1)\)

- **with gain scaling** \(s_0(\cdot) \in \mathcal{C}(\mathbb{R}_{\geq 0}; \mathbb{R}_{> 0})\)

(e.g. to fix minimal gain: \(k_0(t) \geq s_0(t)/\psi_0(t)\) for all \(t \geq 0\))
Control objective and funnel controller \((r = 2)\)

- **objective:** ‘tracking with prescribed transient accuracy’ of \(y_{\text{ref}}(\cdot)\) and \(\dot{y}_{\text{ref}}(\cdot)\)

- **funnel controller** (with derivative feedback)

\[
u(t) = k_0(t)^2 \left( e(t) + \frac{k_1(t)}{k_0(t)} \dot{e}(t) \right), \quad e(t) = y_{\text{ref}}(t) - y(t) \quad \text{(FC}_2)\]

- where \(k_0(t) = \frac{s_0(t)}{\psi_0(t) - |e(t)|}\) and \(k_1(t) = \frac{s_1(t)}{\psi_1(t) - |\dot{e}(t)|}\)

- scaling functions \(s_i(\cdot) \in C(\mathbb{R}_{\geq 0}; \mathbb{R}_{>0}), i = 0, 1\)
Funnel control and steady-state accuracy

- asymptotic accuracy (i.e. $\lim_{t \to \infty} e(t) = 0$), cannot be guaranteed, since $\psi_0(t) \geq \lambda_0 > 0$ for all $t \geq 0$

- typical solution: use of internal model (IM) to achieve asymptotic tracking

\[
\begin{array}{c}
\text{e} \\
\downarrow \text{(FC}_1\text{) or (FC}_2\text{)} \\
\downarrow \\
\text{v} \\
\downarrow \text{(IM)} \\
\downarrow \\
\text{u} \\
\downarrow \text{system} \\
\downarrow \\
\text{y}
\end{array}
\]

- if (IM) has
  - relative degree zero
  - positive high-frequency gain
  - and is minimum-phase

then extended system has identical ‘structural properties’ as system

- standard internal model in industry (for constant signals):
  PI controller

\[
v(t) = u(t) + k_I \int_0^t u(\tau) \, d\tau \\
v(s) = \frac{s + k_I}{s} u(s), \quad k_I > 0
\]
Stiffly coupled servo-systems

- drive torque $m_M(\cdot) \in C(\mathbb{R}_{\geq 0}; \mathbb{R})$ [Nm] (control input)
- load torque $m_L(\cdot) \in L^\infty(\mathbb{R}_{\geq 0}; \mathbb{R})$ [Nm] (disturbance)
- inertia $\Theta > 0$ [kg m$^2$]
- state $\mathbf{x} = (\phi, \omega)^T$: position $\phi$ [rad], speed $\omega$ [rad/s]
- friction (on drive & load side)
- gear with ratio $g_r \in \mathbb{R}\setminus\{0\}$ [1] (neglecting dynamics & backlash)
- signals available for feedback: position $\phi$ and/or speed $\omega = \dot{\phi}$
  (deteriorated by $n_m(\cdot)$ and/or $\dot{n}_m(\cdot)$, resp.)
Speed control implementation

- Standard PI controller
  \[ m_M(t) = k_P e(t) + k_I \int_0^t e(\tau) \, d\tau \]

- PI-funnel controller
  \[ m_M(t) = k_0(t) e(t) + k_I \int_0^t k_0(\tau)e(\tau) \, d\tau \]
  where
  \[ k_0(t) = \frac{s_0(t)}{\psi_0(t) - |e(t)|} \]
Speed control measurement results

Set-point tracking

Reference tracking
Position control implementation

- Standard PID controller (with feedforward)
  \[
  m_M(t) = k_P e(t) + k_I \int_0^t e(\tau) d\tau + k_D \dot{e}(t) + u_F(t)
  \]

- PID-funnel controller
  \[
  m_M(t) = k_0(t)^2 \left( e(t) + \frac{k_1(t)}{k_0(t)} \dot{e}(t) \right) + k_I \int_0^t k_0(\tau)^2 \left( e(\tau) + \frac{k_1(\tau)}{k_0(\tau)} \dot{e}(\tau) \right) d\tau
  \]
  where \( k_i(t) = \frac{s_i(t)}{\psi_i(t) - |e^{(i)}(t)|} \text{ for } i = 0, 1. \]
Position control measurement results

Set-point tracking

Reference tracking

PID-funnel controller

----- PID controller (with feedforward)
Conclusion

To take home

- **funnel control applicable** for speed and position control
- **only structural properties** must be checked (robustness)
- **no** system identification or parameter estimation necessary
- **time-varying gains** (also decrease possible)
- ‘tracking with prescribed transient accuracy’
- **steady state accuracy** in conjunction with PI controller

Some more results (not presented)

- funnel control is also applicable for speed and position control of **elastically coupled servo-systems** (two-mass systems)
- position funnel control of **rigid revolute joint robotic manipulators** is feasible (if inertia matrix is roughly known)
- funnel control in presence of **actuator saturation** is feasible (conservative feasibility condition must be satisfied)
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