Funnel-Control with Integrating Prefilter for Nonlinear Time-varying Two-Mass Flexible Servo Systems

C.M. Hackl, H. Schuster, C. Westermaier and D. Schröder
Overview

1. Funnel-Control (FC)
   - preliminaries, basic idea -

2. Nonlinear Two Mass System
   - block diagram, nonlinearities, auxiliary system -

3. Integrating Prefilter
   - design, stability -

4. Simulation
   - results, comparison with state feedback (SF) -

5. Conclusion & Outlook
Funnel-Control (FC) - Preliminaries

System of Class $S$ - prerequisites:

- relative degree $r = 1$
- minimum-phase (stable zero-dynamics)
- known high-frequency gain $hfg = \text{sgn}(L_g h(x)) > 0$

Controller properties:

- proportional controller
- adaptive time-varying gain
- control law

$u(t) = \alpha \left( \partial F_\phi(t), \Psi(t), \|e(t)\| \right) \cdot e(t) \quad (1)$
Funnel-Control (FC) - Basic Idea

Error $e(t)$, Funnel Boundary $\pm \partial F_\varphi$

Adaption of gain depending on (vertical) distance:

- Short distance $\Rightarrow$ high gain (more aggressive control)
- Great distance $\Rightarrow$ small gain (more relaxed control)

Fig. 2: Error evolution within Funnel Boundary

Time-varying (adaptive) gain $\alpha(t)$ $\Rightarrow$ Boundedness of control error (for sufficiently large but finite control inputs!)

$$\alpha\left(\partial F_\varphi(t), \Psi(t), \|e(t)\|\right) = \frac{\Psi(t)}{d_V(t)} = \frac{\Psi(t)}{\partial F_\varphi(t) - \|e(t)\|} \geq \frac{\Psi(t)}{\partial F_\varphi(t)} \quad \forall t \geq 0 \quad (2)$$

Scaling factor $\Psi(t)$ to choose minimum gain value in advance ("memory effect").
Nonlinear Two-Mass Flexible Servo System (TMS) - System Model

\[
\begin{align*}
\dot{\Omega}_M &= \frac{1}{\Theta_M} \left( M_M \frac{d}{\dot{u}} \cdot \frac{\partial B L(\Delta \phi)}{\partial t} - \frac{c}{\dot{u}} B L(\Delta \phi) \right) \\
\Delta \phi_{BL} &= -M_W N L(\Omega_A) + d \frac{\partial B L(\Delta \phi)}{\partial t} + c B L(\Delta \phi) \\
\end{align*}
\]

Output: \[ y = \Omega_A \]

Fig. 3: Block diagramm - nonlinear Two-Mass-System (converter neglected)
Funnel Control

Nonlinear Two-Mass Flexible Servo System (TMS) - Control Loop

\[ y^* = \Omega_A^* \]

\[ e \]

\[ x_1 \]

\[ K_1 \]

\[ y^* \]

\[ e' \]

\[ u = M_M \]

Nonlinear TMS (see Fig. above)

\[ y = \Omega_A \]

Funnel-Control

Funnel-Control

\[ x_3 = \Omega_A \]

\[ x_2 = \Delta \varphi \]

\[ x_1 = \Omega_M \]

Auxiliary system

**Fig. 4: Block diagramm - Control-Loop with prefilter and Funnel-Control**

Introduction of auxiliary system \((u \rightarrow y')\):

- Reduction of relative degree
- Secure minimum-phase property
- Maintain influence of motor torque \((hfg > 0)\)

\[ K_1 > 0 \]

\[ K_2 > 0 > - \frac{\frac{\partial}{\partial A} \left( K_1 + K_3 \right)}{\Theta_A} \] (3)

\[ 0 > K_3 > -K_1 \]
Prefilter - Design

\[ y^* = \Omega_A \]

\[ e \xrightarrow{} x_i \xrightarrow{} K_V \xrightarrow{} y' \]

\[ e' \xrightarrow{} y' \]

**Fig. 5: Cascaded Loop - Integrating prefILTER with variable gain**

Generation of auxiliary error:

\[ e'(t) = y'^*(t) - y'(t) = K_V(t) \int_{t_0}^{t} e(\tau)d\tau - y'(t) \] (4)

where \( K_V(t) > 0 \)

Auxiliary error starts at:

\[ e'(t_0) = 0 \quad \forall y'(t_0) = 0 \wedge x_i(t_0) = 0 \]

Choice of constant Tolerance Funnel Boundary:

\[ \partial F_\varphi(t) \square \partial F_{const} > 0 \]

(Scaling factor \( \Psi(t) \square \partial F_{const} \) used to guarantee minimum gain of \( \alpha(\square) \geq 1 \) )
Prefilter - Stability

Saturation of control input:  \[ |u(t)| = \alpha(\square) \cdot |e'(t)| \leq u_{\text{max}} \]  (5)

\[ e'(t) = K_v(t) \int_{t_0}^{t} e(\tau) d\tau - y'(t) \leq \frac{u_{\text{max}} \cdot \partial F_{\text{const}}}{u_{\text{max}} + \partial F_{\text{const}}} \]

\[ \Rightarrow \text{Preservation of stability} \] (considering saturation of control input!)

Initial prefilter gain:

Violation of Eq. (5) at time \( t^* \)

\[ K_v^0 = \frac{u_{\text{max}} \cdot \partial F_{\text{const}}}{u_{\text{max}} + \partial F_{\text{const}}} \]  (6)

\[ K_v^0(t^*) = \frac{K_v^0}{2} \]  (7)
Simulation Results - Disturbance Load

Fig. 5a: Load revolution speed for FC & SF

Fig. 5b: Control input (motor torque) for FC & SF

Reference Load Speed: \[ \Omega^*_A(t) = \begin{cases} \frac{10\text{ rad}}{s} & ; t \in [1;8.5 s] \\ 0\text{ rad} & ; \forall t \in [0;10s] \end{cases} \]

Disturbance Load Torque: \[ M_w(t) = 10 Nm \cdot \sigma(t - 5s) \]
Simulation Results - Time-varying Stiffness

Fig. 6a: Load revolution speed for FC & SF

Fig. 6b: Control input (motor torque) for FC & SF

Time-varying stiffness: (negativ ramp)

\[
c(t) = \begin{cases} 
c_0 - 40t & ; 0 \leq t < 9.5s \\
\frac{Nm}{30} & ; t \geq 9.5s
\end{cases}
\]

with \( c_0 = 410 \frac{Nm}{rad} \)

Disturbance Load Torque:

\[
M_W(t) = 10 Nm \cdot \sigma(t - 5s)
\]
Simulation Results - Increased Backlash

**Fig. 7a: Load revolution speed for FC & SF**

**Fig. 7b: Control input (motor torque) for FC & SF**

Reference Load Speed:

\[ \Omega_A^*(t) = 5 \cdot \sin \left( \frac{1 \text{ rad}}{s} \cdot t \right) \]

Increased Backlash:

\[ a_{BL} = 5^\circ \]
Conclusion (achieved goals):

- Stationary accuracy (despite unknown plant!)
- Robustness due parameter deviations or time-varying behavior
- Nonlinearities do *not* affect control performance
- Measurement noise is admissible
- Efficient usage of control input due to adequate weighting of control error (adaption of control gain)
- No Identification/observation and compensation of nonlinearities necessary

Outlook (next steps):

- Expansion of experimental results (laboratory tests)
- Adaptive observers for state estimation