Error Reference Control of Nonlinear Two-Mass Flexible Servo Systems

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Abstract—This paper recapitulates the adaptive (time-varying) control strategy Funnel-Control (FC) and introduces its direct derivative Error Reference Control (ERC) with specially designed Funnel boundaries and auxiliary reference. Both controller designs are comparatively applied to a nonlinear two-mass flexible servo system for speed control. ERC (as derivative of FC) is based on the high-gain controllability of minimum-phase systems with relative degree one and known high-frequency gain. Both implementations allow prescribed transient behavior without identification and/or parameter estimation, although the plant is only structurally known (huge parameter deviations are tolerated). As most industrial applications — also the considered two-mass system — exhibit higher relative degrees, the control strategy is not directly applicable. Therefore a state-feedback like extension is introduced, which assures the reduction of the relative degree and the minimum-phase property. By an additional implementation of a PI-like structure, disturbance rejection and asymptotic tracking of the load speed for a given time-varying velocity trajectory is achieved. Measurement results underpin the achievable performance.

I. INTRODUCTION

In a wide area of practical control tasks in industry, the control engineer has only (very) rough knowledge of the plant — often only the plant structure is obvious. In most applications in mechatronics, nonlinear friction, backlash of gears and damping ratios of mechanical systems are not exactly known or identifiable. Also due to parameter uncertainties of the real plant and without precise system identification (which is often very time-consuming — therefore cost-intensive), linear control strategies reach very easily their limits. In this paper a robust, adaptive (time-varying) control concept is re-examined to bypass all difficulties of linear control design. This control strategy — Funnel-Control — is applicable to a wide class $S$ of systems with relative degree one, a minimum-phase property and a known high-frequency gain [7], [8], [9], [10]. The controller is able to cope with all plants of class $S$, without parameter estimation/identification. Measurement noise and parameter uncertainties are tolerated. This non-identifier based approach enables the control engineer not only to guarantee stability and good tracking performance of the closed-loop system, but also he may predefine the transient behaviour of the plant by a (decaying) limiting function of time, meeting for e.g. customer specifications (if the control input is sufficiently dimensioned) [7]. Funnel-Control adjusts its time-varying proportional gain only by the measured control error and its distance to a predefined limiting function of time — the Funnel Boundary — and therefore absolutely ensures the evolution of the control error within this region, however drastic oscillations and/or overshoots due to the necessary initial Funnel width may occur. In this paper we introduce Error Reference Control — a direct derivative and special design of Funnel-Control, which additionally allows to guide the error evolution along a predefined desired error reference trajectory within a virtual tube (a specially designed asymmetric Funnel Boundary). The special controller design is obtained by combining the overall reference with the desired error reference to an auxiliary reference signal and by the minimal distance evaluation between the measured error and either the upper or lower boundary of the tube, resulting in a correct adaptation of the time-varying proportional gain (similar to that of Funnel-Control). As the system class $S$ restricts the application of both controller designs to most industrial plants (e.g. nonlinear two-mass flexible servo systems) a state-feedback like structure is used which allows to ensure the affiliation of the introduced auxiliary system to class $S$ [6], [15]. The inherent proportional characteristic of both controllers does not allow asymptotic tracking and/or good disturbance rejection, therefore an additional PI-like extension is implemented assuring the convergence of the control error to zero [5].

II. FUNNEL-CONTROL

Funnel-Control, developed by Ichmann et al. [7], is a recent control strategy which is based on high-gain concepts and employs an adjustable proportional (time-varying) gain $\alpha(t)$ to control nonlinear systems of class $S^1$ with relative degree $r=1$, stable zero-dynamics (minimum-phase property) [11] and known high-frequency gain (describing the influence of the control input $u^2$ on the gradient of the output $y$). The system $S$ is governed by the Funnel Controller (see Fig. 1).

![Funnel-Control Block Diagram](image-url)

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1For more explicit information on the system class $S$, the reader is referred to [7], [8], [9], where the subsystems $\sum_1$ and $\sum_2$ with the Operator $T$ are well defined and examples are given for possible systems of class $S$.

2Nomenclature: bold MIMO signals, e.g. $y \in \mathbb{R}^m$; normal SISO signals, e.g. $y \in \mathbb{R}$
the following manner to ensure that the error \( e(t) \)  is continuous and positive function of the error \( e(\cdot) \)  and the euclidian norm \( \| \cdot \| \)  is surrounded by the Funnel. The gain \( \varphi(\cdot) \)  evolves inside the Funnel. For industrial control tasks this holds true for any given reference signal \( y^*(t) \)  and measurable output \( y(t) \). Mathematically this condition may be relaxed by choosing the initial value of the boundary to \( \varphi(0) = \infty \) [7]. Obviously, customer specifications \( (\delta, \tau, \mu) \) — depicted in Fig. 2 — can be easily met by an adequate definition of the Funnel Boundary with \( \varphi(0) = \delta \) and\( \lim_{t \to \infty} \varphi(t) = \mu. \)

### III. ERROR REFERENCE CONTROL

Error Reference Control (ERC) is a direct derivative of Funnel-Control (so far for the SISO case only) and closely corresponds to its source introduced in [7]. Due to its special design of the Funnel Boundary (for the future labelled as „virtual tube“) and adequate choice of its auxiliary reference, Error Reference Control allows not only prescribed transient behavior within given limits (the Funnel region, see Fig. 2) , but also guidance along a desired error reference evolution trajectory (see Fig. 3). The control-loop coincides with that shown in Fig. 1 where the Funnel Controller is now replaced by the Error Reference Controller and all signals by scalar signals. The same class \( S \) of high-gain controllable systems are stabilized and asymptotic tracking within the virtual tube

\[
T_{ERC} = \{ e \in \mathbb{R} | e^*(t) < e < \tau^*(t) \forall t \geq 0 \}
\]  

\(^{4}\delta \) is the accuracy at time \( \tau_{3} \) and \( \mu \) is the desired steady-state accuracy
is guaranteed, if the initial error “starts” within $T_{ERC}$ (see Fig. 3 for a possible and illustrative example tube/boundary design and error evolution). The evolution of the control error is now even more strictly bounded by the virtual tube (see Fig. 3) around a predefined desired error reference evolution — the Error Reference trajectory $e^* \in C^1(\mathbb{R}_{\geq 0}; \mathbb{R})$ — (see Fig. 3 & 4) which is initialized corresponding to the initial error $e(t_0) = y^*(t_0) - y(t_0)$ at time $t_0 \geq 0$, in this paper exemplary set to

$$e^*(t) = \left(\frac{y^*(t_0) - y(t_0)}{e(t_0)}\right) \exp\left(-\frac{t - t_0}{T_{exp}}\right)$$

where its transient evolution may be arbitrarily fixed by the time constant $T_{exp} > 0$. The desired error reference evolution $e^*(\cdot)$ in (obviously) tends to zero, therefore $\lim_{t \to \infty} e^*(t) = 0$. Error Reference Control allows to guide the error along $e^*(\cdot)$ by using following proportional control law (utilizing (3) in the SISO case)

$$u_{ERC}(t) = \alpha_{ERC}(t) \cdot [e(t) - e^*(t)] = \alpha_{ERC}(t) \cdot [y^*(t) - e^*(t) - y(t) - n(t)]$$

with the specially introduced auxiliary reference $y_{ERC}^* = y^* - e^* \in W^{1,\infty}(\mathbb{R}_{\geq 0}; \mathbb{R})$ (corresponding to the SISO case of Funnel-Control [7]) where the time-varying gain

$$\alpha_{ERC}(t) = \frac{\Psi(t)}{\min(\frac{\partial}{\partial t} y(t), \frac{\partial}{\partial t} (y(t)) = 1 + \kappa(t)) =: \partial \bar{F}_{\varphi}(t) = \frac{1}{\bar{\varphi}(t)}$$

and the lower limit

$$\lim_{t \to \infty} e^*(t) = \mu (1 + \kappa(t)) =: \partial \bar{F}_{\varphi}(t) = \frac{1}{\bar{\varphi}(t)}$$

where $\kappa \in C(\mathbb{R}_{\geq 0}; \mathbb{R}_{\geq 0})$ and $\lim_{t \to \infty} \kappa(t) = 0$. The tube $T_{ERC}$ represents a specially designed asymmetric Funnel Boundary with upper $\partial \bar{F}_{\varphi}(\cdot)$ and lower $\partial \bar{F}_{\varphi}(\cdot)$ limit comparable to standard FC. Due to the choice $e^*(t_0) < e(t_0) = e^*(t_0) < \tau^*(t_0)$, the virtual tube will enclose the initial error. Both limits tend to the stationary accuracy

$$\lim_{t \to \infty} \tau^*(t) = \mu$$

and $\lim_{t \to \infty} e^*(t) = -\mu$. In Fig. 4 the sign of the generated control action is shown. Depending whether the error is above or below the desired error trajectory $e^*(\cdot)$, the control input changes its sign and therefore allows appropriate acceleration or deceleration of the system. This property ensures in contrast to standard Funnel-Control design not only the evolution within the virtual tube (or the specially designed Funnel region) but also the possible guidance along the desired predefined of $e^*(\cdot)$.

The following proposal for a damping algorithm has similitively and experimentally shown beneficial effects — namely increased damping (overshoots and oscillations are drastically reduced) — on the system response, but yet the mathematical proof for applicability and general improvement is to follow. The damping algorithm is based on the suggested implementation of a saturated scaling function

$$\Psi(t) = \text{sat} [\Psi_D(e(t) - e^*(t))(\dot{e}(t) - \dot{e}^*(t))]_{0}^{\Psi_{\max}} + \Psi_0$$

with the minimal offset $\Psi_0 \geq 1$, the damper gain $\Psi_D \geq 0$ and the maximal scaling value $\Psi_{\max}$, which limits the damper influence on e.g. noise amplification in the control-loop or the non-continuously (only on a measure of zero!) time derivative of the control error due to the essentially bounded property of the class of reference signals $\tilde{y}^* \in L^{\infty}(\mathbb{R}_{\geq 0}; \mathbb{R})$ [7, 8].

Applications of this algorithm indicate for all made simulations and experiments, that the error will even closer coincide with the desired error reference $e^*(\cdot)$, as any deviation will supplementary increase the control gain $\alpha_{ERC}(\cdot)$ and push the error faster towards the predefined track. Besides that, possible oscillations around $e^*(\cdot)$ are seemingly suppressed as well. The scaling function (17) is obviously also applicable to standard Funnel-Control but is much less
effective — especially at startup (near $t_0 \geq 0$) where the Funnel is still wide — as the control input (1) will only change its sign with the control error.

IV. TWO-MASS-SYSTEM

The considered plant (shown in Fig. 5) is a nonlinear two-mass flexible servo system (2MS), which is a common example for an electrical drive connected to a load machine via gear and flexible shaft. The system has the state vector $x(t)^T = [\Omega_M(t) \Delta \varphi(t) \Omega_A(t)]^T$ with the revolution speed of the motor $\Omega_M$ and of the load machine $\Omega_A$ and the angle of twist $\Delta \varphi = \int_0^t \left( \frac{1}{M} \Omega_M(\tau) - \Omega_A(\tau) \right) d\tau = \frac{1}{M} \varphi_M - \varphi_A$. Its block diagramm is depicted in Fig. 6. The nonlinear state ODE is given by

$$\dot{x} = \begin{bmatrix}
\frac{1}{M} (\dot{\varphi}_M - \frac{d}{dt} \Omega_M(\varphi_A) - \frac{\kappa}{BL} (\Delta \varphi)) \\
\frac{1}{M} (\Omega_M - \Omega_A) + \frac{\kappa}{BL} (\Delta \varphi)
\end{bmatrix}$$

(18)

with the constant parameters $c > 0$ (stiffness of shaft), $d > 0$ (damping coefficient), $\bar{u} > 0$ (gear ratio), $\Theta_M > 0$ and $\Theta_A > 0$ (inertia of the motor and the load, respectively) [13], [14]. The nonlinearities $BL(\cdot)$ and $NF(\cdot)$ represent the bounded backlash of gear with the backlash angle $\Delta \varphi_{BL}(t) := BLC(\Delta \varphi(t))$ and the friction torque $M_{NF}(t) := NF(\Omega_A(t))$ and are modelled continuously (to avoid differential inclusion not covered in [7]) by

$$BLC(\Delta \varphi) = \begin{cases}
\Delta \varphi + a_{BC} & ; \Delta \varphi \leq -a_{BC} \\
0 & ; |\Delta \varphi| < a_{BC} \\
\Delta \varphi - a_{BC} & ; \Delta \varphi \geq a_{BC}
\end{cases}$$

(19)

with the constant half dead zone width $a_{BC} \geq 0$ and

$$NF(\Omega_A) = \frac{M_{NF} \Omega_A}{\pi} + \frac{2 \cdot M_{NF}}{\pi} \arctan(10^3 \cdot \Omega_A)$$

(20)

with the viscous friction slope $M_{NF} > 0$ and the coulomb friction offset $M_{NF} > 0$ [6], [13]. Therefore those do not affect the affiliation of the 2MS to system class $S$ and do not deteriorate the latter control performance of Funnel-Control and Error Reference Control [7], [15]. The current control loop is assumed to be fast enough and is thus neglected (adequate for modern power converters and drives [15]). So the torque $M_M$ can be an arbitrary function of time, whereas the control input

$$|u(t)| = |M_M(t)| \leq M_M^{\max}$$

(21)

is normally constrained by $M_M^{\max} > 0$ to protect the motor and the power electronics unit (converter). Within this paper effects due to saturated control inputs — such as e.g. integrator wind-up — are neglected, as the controller design guarantees not to exceed the given limits. In general, the supplementary use of Saturated Input Compensation would obviate these negative side-effects (not the focus of this contribution) [2], [3].

A. Auxiliary System with relative degree $r' = 1$ and minimum phase property

In [15] it has been shown that the flexible two-mass servo system has relative degree $r_{2MS} = 2$. Therefore the relative degree must be reduced by a state feedback-like structure via an auxiliary output

$$y'(t) = k^T x(t)$$

(22)

resulting in a linear combination of the feedback vector $k^T = [k_1 \ k_2 \ k_3]$ with the state vector $x(t)^T = [x_1(t) \ x_2(t) \ x_3(t)] = [\Omega_M(t) \ \Delta \varphi(t) \ \Omega_A(t)]$. By choosing the feedback structure to $k_1 > 0$, $k_2 > -d/\Theta_A(k_1 + k_3)$ and $k_3 > -k_1$, it can be guaranteed that the relative degree of the 2MS (for any constant parameter set of $\Theta_A$, $\Theta_M$, $c$, $d$, $\bar{u}$ — all greater than 0 by physical means) is reduced to $r_{2MS}' = 1$ and the required minimum phase property is retained [6]. If the third inequality is tightened with $-k_1 < k_3 \neq \frac{\Theta_A}{\Theta_A + k_2}$, observability of the output $y = \Omega_A$ in the auxiliary output $y'$ is assured [15], which is essential for the desired control of the revolution speed $\Omega_A(t)$ of the load machine. As the 2MS is not exactly known, the feedback coefficients may be chosen arbitrarily and independently of the system parameters if following relations

$$k_1 > 0$$

$$k_2 \geq 0$$

$$0 < k_3 > -k_1$$

(23)

5Auxiliary variables are indicated by a prime (‘) symbol.

TABLE I

| $\Theta_M$ | $0.166 \text{ kg} \cdot \text{m}^2$ |
| $\Theta_A$ | $0.333 \text{ kg} \cdot \text{m}^2$ |
| $M_{NF}^{\max}$ | $0.0018 \text{ Nm}$ |
| $M_{NF}^{\max}$ | $0.05 \text{ Nm}$ |
| $d = 0.025 \frac{\text{Nm}}{\text{rad}}$ | |
| $c = 410 \frac{\text{N}}{\text{m}}$ | $M_{BL}^{\max} = 22 \text{ Nm}$ |
| $\bar{u} = 1$ | $M_{BL}(t) = 5 \text{ Nm} \cdot \sigma(t - 5s)$ |

Fig. 5. Laboratory Setup of Nonlinear 2-Mass Flexible Servo System

Fig. 6. Block Diagram of Nonlinear 2-Mass Flexible Servo System
are fulfilled. The known sign of the high-gain frequency re-
 mains untouched by the introduction of the auxiliary system,
as the control input $u(t) = M_M(t)$ and its influence on the
auxiliary output $y'(t) = k_1 \Omega_M(t) + k_2 \Delta \varphi(t) + k_3 \Omega_A(t)$
corresponds to the influence on the original output $y(t)$
(for almost all $t \geq 0$). Recapitulatory, all prerequisites for
the implementation of Funnel-Control and Error Reference
Control at the 2MS are satisfied [6], [15].

B. Control-Loop

In Fig. 7 the control-loop — consisting of either FC or
ERC — is depicted. The $\Pi$ extension represents a simple
nominal PI structure of the form $F_{\Pi}(s) = 1 + \frac{1}{\tau_s s}$
to achieve steady-state accuracy and does not deteriorate the affiliation
to class $S$ [5]. The prefilter $K_V = k_1 + k_3$ is needed to adjust
the reference signal $y'*(t)$ to the inner auxiliary reference signal $y_s'(t)$ [15]. Critical performance issues such as e.g.
integrator wind-up (no occurance/neglected in this paper)
due to constrained control inputs (21) might be overcome by
additional implementation of Saturated Input Compensation
[2], [3].

V. EXPERIMENTAL RESULTS

The goal is to track a given reference velocity signal
$y_*(t) = \Omega_A^*(t)$ with the revolution speed of the load machine
$y(t) = \Omega_A(t)$. The control performances of optimal LQ
State-Feedback$^6$ (SF) [12], Funnel-Control (FC) and Error Reference
Control (ERC) are compared. For all measurements, all three implementations are designed with the
parameters listed in Tab. II & III. The 2MS parameters are
exemplary given in Tab. I (those are only needed for LQ
design!). Sampling time was set to $h = 1 ms$. The reference velocity
$\Omega_A^*(t) = 15 \frac{rad}{s} \cdot \sigma(t) \ldots$
$\sigma(t) \ldots$
$3 \frac{rad}{s} \sin(\frac{\pi}{10}t) \sin(\frac{\pi}{5}t) \cdot \sigma(t-16s) \ldots$
$3 \frac{rad}{s} \sin(\frac{\pi}{10}t) \sin(\frac{\pi}{5}t) \cdot \sigma(t-20s)$

$\sigma(t) \ldots$
$3 \frac{rad}{s} \sin(\frac{\pi}{10}t) \sin(\frac{\pi}{5}t) \cdot \sigma(t-20s)$

is generated by an initial step, a ramp and finally a super-
position of two sinusoidal signals (see red line in Fig. 8).

The measurement results depicted in Fig. 8 (Comparison of
Velocities in Fig. 8(a) and Errors in Fig. 8(b)) clearly show the good tracking performance of both adaptive (time-

to VAR) designs FC & ERC, whereas SF exhibits large
contouring errors — especially when tracking ramp signals
(see Fig. 9(a)) or sinusoidal signals (see Fig. 9(b)). All
three controllers achieve good disturbance rejection, whereas
ERC reacts on the load torque $M_M(t) = 5Nm \cdot \sigma(t-5s)$
with the smallest deviation (see Fig. 9(a)). Although, ERC is
designed without knowledge of the system parameters,
especially at the beginning (tracking the step $0 \ldots 5s$) a
nearly as smooth transient response as with SF is achieved.

Here standard FC exhibits the undesirable but inevitable
overshoot (see Fig. 8(a) or 8(b)). The auxiliary errors $e'(t)$
of both FC and ERC remain within their predefined regions,
the Exponential Funnel $F_{exp}$ and the virtual tube $T_{ERC}$
for all time, respectively (see Fig. 11). Therefore an adequate
adaptation of the control gains (see Fig. 10(a)) is needed, here
the ERC control gain is additionally scaled by the algorithm
proposed in Eq. (17) resulting in a better damped system
response. In conclusion, due to effective adjustment of the
control input sign (see Fig. 10(b), smoother and only positive

$^6$Designed by the optimal quality function $J$ with stability margin $\alpha = 3$
$\min K_r J = \int_0^\infty \exp(2\alpha x(x^2 Q x + r \cdot u^2)) \, dx$ (Robust Design).
$\gamma \sigma(t-t_0) = \begin{cases} 0, & t < t_0 \\ 0.9, & t \geq t_0 \end{cases}$

$^8$This value represents the integrator gain $K_I$ of the outer loop of SF with
integrating prefilter
Fig. 8. Comparison of Velocity Tracking Performance with Load $M(t) = 5N \cdot m \cdot \sigma(t - 5s)$ (Measurement)

(a) Reference & Velocities

(b) Velocity Errors

Fig. 9. Zooms: Comparison of Velocity Tracking Performance (Measurement)

(a) Zoom I (Disturbance Load & Start of Ramp)

(b) Zoom II (End of Ramp & Start of Sinusoidal)

Fig. 10. Comparison of Control Gains and Control Inputs (Measurement)

(a) Control Gains

(b) Control Inputs
in contrast to FC), ERC ensures quite nice guidance along the error reference trajectory without any initial overshoot.

VI. CONCLUSION

In the presented paper, a nonlinear two-mass flexible servo system was controlled by the adaptive (time-varying) control scheme Funnel-Control, which neither identifies nor estimates the system parameters. Two controller designs of this control strategy — standard Funnel-Control and the newly introduced Error Reference Control — are examined and compared. In order to allow the application of both implementations, only the system structure must be known and fit the three conditions: relative degree one, minimum-phase & positive high-frequency gain. The considered system exhibits relative degree two, therefore the relative degree of the plant must be reduced by a state-feedback like extension, assuring its affiliation to the class of high-gain controllable systems. To guarantee accurate asymptotic tracking under load, nonlinear friction and backlash a nominal PI-like structure is implemented. Both adaptive implementations yield enhanced tracking performance compared to standard LQ state feedback control and tolerate measurement noise. Whereas standard Funnel-Control exhibits large initial overshoot (and oscillation) of the error within the Funnel region, its new derivative Error Reference Control smoothly guides the system along a prescribed and desired error reference trajectory within a specially designed asymmetric Funnel Boundary (the virtual tube), additionally suppressing oscillations and overshoots by adequate gain scaling. Measurement results show the industrial applicability of both designs of the time-varying control concept and underpin the achievable tracking performance — especially that of Error Reference Control, although the system parameters need not to be known for control design.

REFERENCES