I. Introduction

Dielectric electro-active polymers (EAP) are lightweight, inexpensive, fracture tolerant, pliable, easily configurable and have a significant deformation response to electrical stimulation [1]. The attractive characteristics offer the potential of these materials being suitable for product-integrated actuators. Current models [1-8] do not fully integrate all of the physical domains needed to design actuator systems. A model which fully integrates the multiple physical domains but retains the electrostatic, mechanical, and mechanical models would substantially facilitate design.

This paper presents such a multi-physics model for EAP actuators, combining the electrostatic properties of the electrical circuit with the physical properties of compliant electrodes; of the polymer material. The capabilities of the developed model are discussed and verified with lab experiments on test EAP actuators.

II. Motivation

The advance of technology increases demand for actuators that are highly integrated and compact. In many instances, traditional electric motors and solenoids may be difficult to implement in the application. In the extreme cases, MEMS and medical instruments are samples of the system that have significantly different requirements on the actuator. Power assists devices that increase the mobility of a physically impaired person require an actuator that performs similar to biological muscle. In the domestic application, the performance of humanoid robots is still limited by their actuators. A number of these humanoid robots have been introduced by automobile companies such as Toyota and Honda. One can expect their performance should increase dramatically when they are fully commercialised. In the military application, human amplifier and exoskeleton are under intensive research as a soldier’s survivability takes priority. These actuators take some common attributes: light weight, high flexibility, easily integrated and configurable, inexpensive, fracture tolerant, and pliable. A number of these properties are commons in polymers.

EAP have attracted many attentions as it has been shown to be a potential actuator material. Currently, no commercial actuator based on EAP available. The underlying physics is still not well understood. Still, a physics-based model will aid the analysis of the actuator and the development of the controller, and gives some insights to the problem where engineering decisions can be made.

II. Nonlinear Model of Polymer Actuator

A. Electrical Circuit of the Actuator

The pre-strained polymer with applied compliant electrodes on the top and bottom side can be considered as a capacitance, $C_p(x_1,x_2,x_3)$, depending on its geometry. Furthermore the compliant electrodes and the special high voltage cables can be treated as a serial connection of resistances $R_p+R$ and an inductance $L_p$. The electrical circuit is depicted in Fig. 1.

![Fig. 1 Electrical circuit of polymer with compliant electrodes](image-url)

With Kirchhoff’s Voltage Law the circuit can be described by

$$e_i(t) = L_p \frac{di}{dt} + (R_p + R) i(t) + e_c(t)$$  \hspace{1cm} (1)

Substituting
\[ e_i(t) = \frac{1}{C_p(x_1,x_2,x_3)} \int e_i(t) dt = \frac{1}{C_p(x_1,x_2,x_3)} q(t) \]

and defining the system state vector as
\[ x_1 = \begin{bmatrix} i \\ q \end{bmatrix} \]
thus, the state space equations can be given by
\[ \frac{di}{dt} = \frac{1}{L_p} \left( -\left( R_p + R \right) i - \frac{1}{C_p(x_1,x_2,x_3)} q + e_i \right) \]
and
\[ \frac{dQ}{dt} = i \]

The capacitance, \( C_p(x_1,x_2,x_3) \), will change, when the dimensions (length \( x_1 \), width \( x_2 \), thickness \( x_3 \)) of the polymer alter by an external load. The electrical circuit can be transformed into a block diagram.

\[ \text{Figure 1. Electrical circuit of the EAP actuator.} \]

Following the definition of \( \lambda_i = \frac{x_i'}{x_i} \) in figure 2, where \( x_i \) is the original length and the prime symbol denotes the deformation length, with the assumption of incompressibility of the polymer material [1,2,3,4,10] giving
\[ \lambda_1 \lambda_2 \lambda_3 = 1 \] \hspace{1cm} (5)

\[ \text{Figure 2. General configuration of a polymer under stress.} \]

If one allows \( \lambda_2 = 1 \) by the configuration of the actuator, thus letting
\[ \lambda_1 = \frac{1}{\lambda_3} = \lambda, \] \hspace{1cm} (6)
and assume the polymer is the dielectric with the parallel plates forming on the x-y plane, then \( C_p \) can be computed by
\[ C_p = \frac{\varepsilon \varepsilon_0 A}{d} = \frac{\varepsilon \varepsilon_0 \lambda_3 x_1 \lambda_2 x_2}{\lambda_3 x_3} = \frac{\varepsilon \varepsilon_0 \lambda^2 x_1 x_2}{x_3} \] \hspace{1cm} (7)
which is a function of \( \lambda \).

**B. Induced Pressure and Traction on the Actuator**

When charging the compliant electrodes, the unlike charges attract each other and induce a (negative) pressure \( \sigma_{\text{maxwell}} \) perpendicular to the area of the compliant electrodes [5,6,10]
\[ \sigma_{\text{maxwell}} = -\varepsilon \varepsilon_0 \left( \frac{1}{x_1} \right)^2 Q. \] \hspace{1cm} (9)

Given that \( Q = C_p V \), a quick change of variable by invoking equation (7) becomes
\[ \sigma_{\text{maxwell}} = -\frac{1}{\varepsilon \varepsilon_0} \lambda^2 \left( \frac{1}{x_1 x_2} \right)^2 Q. \] \hspace{1cm} (9)

This change is necessary since the state variables are as in (2). The accumulation of charge on the capacitor plates will produce a pressure force. With the plates being the compliant electrodes, the electrostatic pressure will force the polymer actuator to mechanically contract in thickness and expand in area as depicted in Fig. 3.

\[ \text{Figure 3. The configuration of EAP actuator with } \lambda_2 = 1. \]

**C. Dynamics and Neo Hookain Model of the Actuator**

When subjected to modest loads, most solids obey Hooke’s Law, which is a linear elastic model. This model quickly fails when the strain becomes large. In the case of EAP material, the elastomer can deform as much as 800% in laboratory conditions. One of the features of the elastomers is that they can undergo large deformation from which they fully (or almost fully) recover, thus no permanent deformation. There exists standard models such as Neo-Hookean and the Mooney-Rivlin model utilizing the strain energy function (SEF) and/or finite strain (SF) theories and a convolution relationship between stress and strain rate. The analysis of the actuator quickly transform into a plane-stress analysis.

The configuration or deformation of the actuator will be depending on the balance of stresses. Define the external load stress
\[ \sigma_{\text{load}} = \frac{F_L}{\lambda_3 x_2 \lambda_3 x_3} = \lambda^2 \frac{F_L}{x_1 x_2 x_3}, \] \hspace{1cm} (10)
where \( F_L \) is the external load force. Formulating the problem by putting the \( \sigma_{\text{maxwell}} \) on the x-y plane, and the external load stress \( \sigma_{\text{load}} \) on the y-z plane, one will obtain the problem in a plane stress configuration.
The deformation is also related to the material coupled by the incompressibility of the material. The relations suggest that the stresses are expressed as a special case of an uniaxial loading under the Neo Hookain material, extending the analysis one can compute the stress on each surface.

Following the Toni decomposition of a plane-stress problem, making use of the deformation relation $\lambda_n$, defining the configuration of the actuator $A$ as

$$A = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

and formulating the nominal stress $S = JA^{-1}T$, where $J = \det(A) = 1$ by the assumption of equation (5), and the stress matrix $T$ is defined by the conservation of linear momentum as

$$T = \begin{bmatrix} G \lambda_1^2 - p & 0 & 0 \\ 0 & G \lambda_2^2 - p & 0 \\ 0 & 0 & G \lambda_3^2 - p \end{bmatrix},$$

where $p$ is a hydrostatic force and $G$ is the shear modulus. Equating $S$ (by components) with the applied stress, giving

$$\frac{1}{\lambda_1^2}(G \lambda_1^2 - p) = \sigma_{\text{load}} \tag{10}$$

$$\frac{1}{\lambda_3^2}(G \lambda_3^2 - p) = \sigma_{\text{maxwell}}. \tag{11}$$

The relations suggest that the stresses are coupled by the incompressibility of the material. The deformation is also related to the material constant $G$.

One observation is made that even though the material can sustain large deformation, there is an operation limit at about 300%. Beyond 300% strain, an extremely large stress has to be applied to obtain further deformation. Following the kinetic theory of rubber elasticity [11] which is a special case of an uniaxial loading under the Neo Hookain model, the stress-strain relation can be expressed as

$$\sigma = G_0\left(\lambda + \frac{l}{\lambda}\right)$$

Figure 5 shows this stress-strain relationship. The interpretation is that when a compression (negative) stress is applied, a substantially increases of stress is required beyond $\lambda = 0.3$. In other words, when the $\lambda_j \geq 0.3$, a dramatic increase of $\sigma_{\text{maxwell}}$ is needed to give further extension in $\lambda_j$.

Figure 5. Neo-Hookean model. When $\lambda < 1$, a compression force is required. At $\lambda \geq 0.3$, a dramatic increase of the stress suggests a theoretical limit to the maximum deformation.

D. Combined Nonlinear Model and State Block Diagram

Combining equation (10) and (11) and substitute in equation (6), one can arrive at the following stress balance relation

$$h(Q,F_L) = G\left(-\lambda^3 + \frac{l}{\lambda}\right) + \lambda^2 \frac{F_L}{x_2 x_3} + \frac{1}{\epsilon_0} \frac{l}{\lambda^2} \left(\frac{1}{x_1 x_2}\right)^2 Q^2. \tag{12}$$

which relates the extension ratio $\lambda$, charge $Q$, and $F_L$. The material elasticity term $\frac{1}{\lambda_3^2}(G \lambda_3^2 - p)$ remains in the equation. Thus when $h(Q,F_L) = 0$, giving the balance between $\sigma_{\text{load}}$ $\sigma_{\text{elastic}}$, and $\sigma_{\text{maxwell}}$, there exists a real and positive $\lambda$ such that this equation holds. Therefore, at a given charge and external load, there exists a configuration such that the stresses are at equilibrium. A state block diagram can be used to reveal more insights.

Figure 6. The non-linear state block diagram of the relation between the stresses

Noted that this is not in the standard state block diagram format since the terms are non-linear and the cross-coupled feedbacks are not shown in the standard representation. However, as
suggested by equation (10) and (11), the stresses are coupled by the incompressibility, thus the feedback of $\lambda$ can be seen in all three blocks. The diagram also suggests that the $\sigma_{\text{elastic}}$ is caused by $\sigma_{\text{maxwell}}$ and $\sigma_{\text{load}}$. If both of them are zero, then $\lambda = 1$ which is the original undeformed configuration. If the physical parameters in the non-linear state block diagram are known, one can produce the following relation

$$h(V, F_L) = G(-x^3 \frac{1}{\lambda} + \lambda^2 + \lambda^3 \frac{F_L}{x_2 x_3} + \lambda^2 \epsilon \epsilon_0),$$

Solving $h(V, F_L)$ for $V$ and rewriting as

$$V = f(\lambda^*, F_L),$$

where $\lambda^*$ is the desired extension, one obtain $V$ as a function of $\lambda^*$ and $F_L$. Noted that because of incompressibility, if $\lambda^* < \lambda$ where $\lambda$ is already determined by $F_L$ at the static state, then $V$ will be complex which implies that $\lambda^*$ is not achievable.

### III. Parameter Estimation

For observations and measurements the 3M VHB 4905/4910 Tapes with compliant electrodes made out of carbon black dust were used as shown in figure 9.

![Figure 7](image_url)  
**Figure 7.** The relation between the extension ratio $\lambda$, external load $F_L$, and the applied voltage $V$. A completed non-linear state block diagram is shown in figure 8. Noted that $C_p$ has been replaced by equation (7), and the output $x$ is the actual extension of the actuator. When a voltage is applied, charges are built up on the actuator causing the deformation.

#### E. Command feedforward realised by the analytical model

Command feedforward (CFF) can be used to improve command tracking, thus the close-loop controller is primary for improving disturbance rejection. CFF can be realised if the physical model of the system is known or well approximated. Since the electrical dynamics is much faster than the mechanical dynamics, following the same analysis on the mechanical dynamics but without variable substitution as in equation (8), one obtains a different form of equation (12) as

$$h(V, F_L) = G(-\lambda^3 + \frac{1}{\lambda} + \lambda^2 \frac{F_L}{x_2 x_3} + \lambda^2 \epsilon \epsilon_0 V^2).$$

Solving $h(V, F_L)$ for $V$ and rewriting as

$$V = f(\lambda^*, F_L) = \frac{\sqrt{x_3 \sqrt{x_3} \frac{F \lambda^2}{x_2 x_3} - \frac{1}{2} G \left(\frac{1}{\lambda^* - \lambda^2}\right)}}{\lambda^* \sqrt{\epsilon \epsilon_0}},$$

where $\lambda^*$ is the desired extension, one obtain $V$ as a function of $\lambda^*$ and $F_L$. Noted that because of incompressibility, if $\lambda^* < \lambda$ where $\lambda$ is already determined by $F_L$ at the static state, then $V$ will be complex which implies that $\lambda^*$ is not achievable.

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![Figure 9](image_url)  
**Figure 9: Planar Elastic Polymer Actuator in Test Stand**

The parameters of the model were estimated or measured and are given in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_p$</td>
<td>500 KΩ</td>
</tr>
<tr>
<td>$R$ (negligible)</td>
<td>20 KΩ</td>
</tr>
<tr>
<td>$L_p$</td>
<td>1.3H</td>
</tr>
<tr>
<td>$M$</td>
<td>70 g</td>
</tr>
<tr>
<td>$\epsilon_r$</td>
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<tr>
<td>$E$</td>
<td>96 kN/m²</td>
</tr>
<tr>
<td>$x_1$</td>
<td>35 mm</td>
</tr>
<tr>
<td>$x_2$</td>
<td>31 mm</td>
</tr>
<tr>
<td>$x_3$</td>
<td>0.5 mm</td>
</tr>
</tbody>
</table>

**Table 1:** Parameters of the actuator/model

### IV. Analytical and Experimental Results

The developed model was used to evaluate the design and behaviour of the EAP actuator. The analytical results were compared with the
qualitative observed behaviour of the actuator. The model is capable to predict the correct system response to several excitation voltages $V$.

A. Voltage-Displacement Relationship

The voltage-displacement derived from the model is similar to the observed system response. A fixed load is applied to the actuator to simulate a constant applied force.

![Graph showing Voltage vs. Displacement](image1)

Figure 10. Plot of the Voltage vs. Displacement. Blue: experiment result. Orange: simulation result. Green: simulation result correction factor.

The model overestimates the response of the actuator due to the assumption of $\lambda_2 = 1$ is not fully satisfied by the crude fabrication of laboratory specimen. The growth rate of $\lambda$ should be reduced by $\frac{1}{\sqrt{2}}$. Also, the data from the region near 2500V is not accurate due to the noise from the DC to high-voltage DC converter. Thus, the data at that region does not truly reflect the behaviour of the actuator.

B. Deformation/Displacement

The model estimated the actuator extension under various load and results were compared with the experiment data. A series of external load at 0.1N increment with 0.5N initial load were applied to the actuator and a 3000V step voltage was then applied. The results show some relaxation and also inaccuracy of the model at the extreme limit. Figure 11 shows the analytical and experimental results.

This relaxation phenomenon has been reported in [12]. The inaccuracy can be explained by the validity of the assumptions. With the increase of the external load causing the actuator to deform “out of shap,” thus the geometry assumptions were no longer holds. The model begins to underestimate the extensions but it still can describe the general behaviour of the maximum strain limit.

C. Non-linear Polymer Behaviour

It is a known fact that viscoelastic materials often exhibit hysteresis. This can be observed during cyclic loading of the EAP material. The model developed in the pervious section is a static strain model without considering the energy dissipations and other factors, thus it does not describe the time-dependent behaviours of the actuator.

![Graph showing Cyclic response of the EAP actuator](image2)

Figure 12. Cyclic response of the EAP actuator.

The developed model was then implemented in MATLAB/SIMULINK and simulations were run with the introduced parameter values from pervious section. To include the effects of the hysteresis, a Kelvin viscoelastic model was included. The Kelvin viscoelastic model is a simple mechanical damper in parallel with a spring to augment the effect of the time-dependent behaviour. An extensive list of model exists, such as the standard linear model which is used in finite element tools such as FEMLAB.

![Diagram of Kelvin and Standard Linear models](image3)

Figure 13. Left: Kelvin model. Right: Standard Linear model.

A few things are observed form the MATLAB simulation shown in figure 14. First is that the magnitude of the response is smaller. This is obvious by the fact that the Kelvin model reaches an intermediate state thus the absolute magnitude should be smaller. This also shows a limitation of using the Kelvin model in series with the analytical model.
VI. Conclusion

The model presented integrates the multi-physics properties inherent in electro-active polymer actuators. It retains the physical attributes needed for design and modelling for control. It helps to understand the highly non-linear behaviour of EAP actuators stimulated by charges.

To properly describe the time-dependent behaviour of the material used by the actuator, it is possible to model the system in FEMLAB, and import it into MATLAB/SIMULINK. The parameters required for the viscoelastic model can be obtained by Dynamic Mechanical Analyser (DMA). The viscoelastic model will be a standard linear model and it will have additional dynamics that are not included in the Kelvin model.

The experimental results also suggest some design and operation parameters if EAP were to be used as an actuator. Hysteresis and other time-dependent properties can be overcome by better material engineering.

References


Figure 8. The non-linear state block diagram of the EAP actuator.

Figure 11. Experimental and analytical results. Since the deformation at the extreme cases deforms the actuator dramatically, some of the assumptions used in the model are no longer valid thus giving errors.